Sharing the Surplus Generated from Noncooperative Cost Sharing: The Case of Nonpoint Associations and Water Quality Trading*

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Abstract

This paper examines how a regulatory authority might subsidize (i.e., cost share) the participation of associations (or teams) of agents in a surplus-generating economic activity, and how the agents might in turn cooperatively share the surplus. Toward this end, a subgame-perfect equilibrium concept is used to model the "multilateral contracting" relationship between the regulatory authority and the associations when the authority has incomplete information about both the association's behavior and the natural environment. A common surplus-sharing rule – the Shapley value – is then applied to model the relationship among agents comprising a given association. We show that for convex surplus-sharing games the Shapley Value is included in a non-empty core. The analysis depicts the relationship between a federal regulatory agency and associations of nonpoint pollution sources in a watershed-wide water quality trading market.

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1 Introduction

The efficacy of incentives contracts for resolving adverse selection and moral hazard problems in the procurement process is by now well understood. Generally speaking, the principle pays the agent(s) an informational rent in order to procure a desired (typically a socially optimal) outcome (Laffont and Tirole, 1993). In the case of multilateral contracting, e.g., a single principle contracting with two or more agents, the procurement problem extends beyond controlling a single agent's decision problem to designing a game that accounts for the agents' interactions with each other (Bolton and Dewatripont, 2005).[1] Factors complicating these types of problems include asymmetric (or hidden) information, symmetric (or common) uncertainty regarding nature, and the absence of a unique dominant strategy equilibrium. In cases where heterogeneous agents are capable of coalescing into a smaller number of teams (or associations), the complexity of the contracting problem is reduced. However, the question now arises as to how an association might best share any surplus generated from the game. At the very least the sharing rule should result in a core allocation, as this maintains a consistency between the association's decision to have formed in the first place and then to have chosen the particular sharing rule.

This paper considers a unique multilateral contracting problem, where (1) the associations have hidden information about their abatement costs and (2) both the principle and associations lack information about nature, i.e., the environment within which they are interacting. We adopt the subgame-perfect equilibrium concept to solve a two-stage game between the associations and the principle, or regulator. In the first stage of the game, the regulator chooses a cost-share parameter and an initial resource endowment for each association. Given the regulator's choices, the associations then choose their resource allocations and how much of the resource to trade in the second stage of the game. At the conclusion of the second stage, each association determines how to share its surplus from trading.

Perhaps the best example of this type of game is the idealized control of nonpoint source water pollution through water quality trading (WQT).[2] Since the passage of the 1972 and 1977 Federal Water Pollution Control Act Amendments, approximately 34,000 of the nation's water bodies

[1]For a concise overview of the mechanism design problem see Mas-Colell, et al. (1995, Chapter 23).

[2]We say "idealized" because to date few WQT markets have been established for watersheds that include predominantly nonpoint sources (King, 2005).

have either remained or become noncompliant with the amendments' standards for drinking water, contact recreation, or aquatic life support (EPA, 2005). The main factor contributing to this noncompliance is the "loading" of nutrient- and pesticide-based pollutants from agricultural non-point sources (NPSs), e.g., crop and feedlot operations, through natural runoff and leaching processes (Freeman, 2002). Regulation of NPSs has been stymied by the very nature of the loadings themselves - they are diffuse, which effectively obviates the ability to monitor and thereby distinguish which loadings belong to which NPSs within a given watershed. In other words, NPS loadings typify symmetric uncertainty regarding nature.

A number of theoretical and practical solutions have been proposed for handling this type of uncertainty. Among the more recent is WQT. WQT is based on the notion that sources within a given watershed (both point and non-point) face different abatement cost structures. Assuming these cost differences are wide enough, and that a given source's abatement level can be estimated with a reasonable amount of certainty, the overall control costs of meeting a given watershed-wide pollution standard can be minimized if those sources facing relatively low control costs effectively over-abate and sell to relatively high-cost sources their excess abatement "credits" via a formal WQT market. To date however, the involvement of NPSs in WQT has been virtually non-existant, even in spite of the National Resource Conservation Service's (NRCS) generous best management practice (BMP) subsidy program, known as the Environmental Quality Incentives Program (EQIP).[3]

This paper offers one possible solution to the NPS-loading problem in the presence of WQT, by examining how a regulator (e.g., the NRCS) might ideally subsidize the BMPs of different associations (or teams) of NPSs in the presence of a watershed-wide WQT market.[4] The subgame-perfect equilibrium concept is used to model the relationship between the NRCS and NPS associations.[5] A

[3]EQIP was reauthorized in the Farm Security and Rural Investment Act of 2002 to *inter alia* provide financial and technical assistance in support of BMP implementation on eligible agricultural land. EQIP cost-shares generally range between 75 and 90 percent of the total cost of BMP implementation (NRCS, 2006).

[4]As demonstrated in Section 2, our solution is indeed "ideal" as it requires (i) enough information about individual NPS abatement costs within a given watershed to estimate continuous marginal cost functions, (ii) individual marginal cost functions that are both derangible and aggregatable to association-level cost functions, and (iii) enough information about the watershed's topography, soil types, and hydrology to estimate NPS loadings and spatial equivalence ratios (or transfer coefficients). Breetz, et al. (2005) suggest that coalescing NPSs into associations may increase trust and communication among the association's members, which in turn leads to increased participation in WQT. In an experimental setting, Vossler, et al. (2006) have shown that "cheap talk" communication can actually induce association members to over-correct their loadings. In addition, the creation of associations can greatly reduce the number of different entities for which the NRCS must determine subsidy rates, thus enhancing the efficacy of administering the BMP subsidy program.

[5]See Caplan, et al., 2000 and Wellisch, 1994 for examples of this concept in the public goods literature.

standard surplus-sharing rule – the Shapley Value – is then applied (e.g., Moulin, 1988 and 2004) to model the relationship among NPSs within a given association.[6] The goal here is to understand how an association might coordinate (and reward) abatement effort among its member NPSs as a result of their participation in a WQT market.

Although the economic literature on NPS pollution is quite extensive, it has until recently focused exclusively on the use of penalty schemes such as taxation and fines that account for the uncertainty of NPS loadings.[7] A nascent strand of this literature has also investigated the properties of WQT as a potential NPS control mechanism.[8] The main focus of this literature has been to identify optimal trading ratios between NPSs and point sources that incorporate uncertainty associated with the NPS loadings. To our knowledge, the role of NRCS subsidization of BMPs in fostering an efficient allocation of NPS abatement effort via WQT has yet to be explored.

We find that cost-sharing in concert with WQT can incentivise associations to independently abate at "constrained-efficient" allocations in the presence of uncertainty regarding their loadings. The extent to which the efficient allocation is constrained by both the asymmetric information concerning the associations' costs and uncertainty about the physical characteristics of the watershed (in terms of the spatial differences among NPSs that distinguish the potency of their respective loadings) is demonstrated both analytically and numerically. Conditions are then derived under which the Shapley Value can be used to optimally divide the WQT surplus among the members of an association.

The next section lays out the basic contracting model describing the interaction between the NRCS and the NPS associations, which in turn interact with each other and with point sources in a WQT market. Analytical solutions to the model are provided for both the full-information benchmark and the case of incomplete information about NPS loadings on the part of the NRCS. Because the analytical solutions are cumbersome, Section 3 solves the model numerically in order to provide estimates of the relative values of the key variables involved. Section 4 characterizes the Shapley value, a common rule that might be adopted by an association to derive a core allocation,

[6] In this case, "surplus" refers to the potential profit that an association might obtain by participating in a WQT market.

[7] Good overviews of this literature are provided in Shortle and Horan (2001) and Tomasi, et al. (1994).

[8] As Shortle and Horan (2001) point out, NPS participation in a WQT market can be based on point-source emissions for NPS input trading or point-source emissions for NPS estimated-loadings trading. In this paper, we focus on the latter. More recent papers related to Shortle and Horan (2001) include Shortle and Horan (2005) and Farrow, et al. (2005).

i.e., a sharing of the profit attained through participation in a WQT market that induces each member of the association to voluntarily remain a member of the association. Section 5 summarizes the paper's main findings.

2 Cost Sharing in the Presence of Water Quality Trading

2.1 The Case of Full Information

For ease of exposition, we assume that a watershed's NPSs are coalesced into two separate associations, denoted $A1$ and $A2$. There also exists one point source, denoted P. $A1$ and $A2$ interact with the NRCS and with P in the following two-stage game. In the first stage, the NRCS sets the associations' cost-share rates α_1 and α_2, $0 < \alpha_j \leq 1, j = A1, A2$, as well as the initial (abatement) allocations $\bar{a}_{A1}$ and $\bar{a}_{A2}$ for $A1$ and $A2$, respectively, and $\bar{a}_P$ for P.[9] In the second stage, the associations and point source (henceforth "agents") engage in competitive WQT by selecting their respective abatement effort levels a_{A1}, a_{A2}, and a_P, respectively (and thus their abatement credits available for sale(in need of purchase), $a_j - \bar{a}_j > (<) 0, j = A1, A2$, and P, whichever the case may be). As a result of "moving" after the NRCS, the associations take their respective subsidy rates and initial allocations as given (as does the point source with respect to its initial allocation). However, by moving first the NRCS is able to anticipate the effects that its choice of the subsidy rates and initial allocations will have on the agents' subsequent abatement effort (i.e., the agents' respective "response functions") and the per-unit price of abatement credits, ρ. In a subgame perfect equilibrium, the NRCS correctly anticipates these response functions.

A subgame perfect equilibrium is solved through backward induction. We therefore begin our analysis with the second stage of the game, i.e., the WQT market. In a WQT market, Association j takes α_j, $\bar{a}_j$, and ρ as given and chooses its abatement effort level a_j to minimize its cost of

[9] In reality, the NRCS is unlikely to be responsible for setting the initial allocations for WQT. Instead, this responsibility will lie with state department(s) of environmental quality (DEQs). In the context of our model, the regulatory authority might therefore best be interpreted as a joint agency, incorporating both the NRCS and DEQ. This joint agency fully characterizes the relationship between NPS polluters and the regulatory authority.

participating in the market, i.e.,[10]

$$\min_{a_j} \quad \alpha_j C_j(a_j) - \rho\tau_j[a_j - \bar{a}_j], \quad j = A1, A2$$

where $C_j(a_j)$ is j's (total) abatement cost function (increasing and convex in a_j) and τ_j is j's corresponding spatial equivalence ratio (i.e., transfer coefficient).[11] This problem results in the familiar optimality conditions,

$$\alpha_j \left[\frac{\partial C_j}{\partial a_j}\right] = \rho\tau_j, \quad j = A1, A2 \tag{1}$$

The point source P similarly chooses its abatement effort level a_P to minimize its cost of market participation,

$$\min_{a_P} \quad C_P(a_P) - \rho\tau_P[a_P - \bar{a}_P]$$

resulting in the corresponding optimality condition,

$$\frac{\partial C_P}{\partial a_P} = \rho\tau_P. \tag{2}$$

Lastly, market price ρ is determined via the market-clearing condition,

$$\bar{A} = \tau_{A1}\bar{a}_{A1} + \tau_{A2}\bar{a}_{A2} + \tau_P\bar{a}_P \leq \tau_{A1}a_{A1} + \tau_{A2}a_{A2} + \tau_P a_P \tag{3}$$

where $\bar{A}$ is a receptor point's pre-determined concentration of the targeted pollutant, e.g., as specified in a Total Maximum Daily Load (TMDL). Invoking the implicit function theorem, the associations' equilibrium abatement efforts and market price, respectively, may be written as $a_{A1} \equiv a_{A1}(\alpha_1, \alpha_2)$, $a_{A2} \equiv a_{A2}(\alpha_1, \alpha_2)$, $\rho \equiv \rho(\alpha_1, \alpha_2)$.[12]

In the first stage, the NRCS chooses the associations' respective cost-share rates α_j and initial

[10] Parentheses denote a functional relationship, while square brackets indicate the application of a given operation on the included variables.

[11] $C_j(a_j)$ therefore represents a continuous aggregate total cost function. It is well-known that an aggregate of convex functions is itself convex (e.g., Simon and Blume, 1994, pg. 519).

[12] For expository purposes, the transfer coefficients $\tau_j, j = A1, A2$, and P, and TMDL concentration $\bar{A}$ are subsumed in these implicit functions. The initial allocations $\bar{a}_j, j = A1, A2$, and P affect neither market price ρ nor abatement effort (only the agents' respective profitabilities). The abatement effort of point source P is not explicitly mentioned, as it neither figures in nor impacts the ensuing first-stage analysis.

allocations $\bar{a}_j$ to minimize its own cost-share obligation,

$$\min_{\alpha_j, \bar{a}_j} \quad [1-\alpha_1]C_{A1}(a_{A1}(\alpha_1,\alpha_2)) + [1-\alpha_2]C_{A2}(a_{A2}(\alpha_1,\alpha_2))$$

subject to (**??**) and,

$$\tau_{A1}\rho(\alpha_1,\alpha_2)[a_{A1}(\alpha_1,\alpha_2) - \bar{a}_{A1}] - \alpha_1 C_{A1}(a_{A1}(\alpha_1,\alpha_2)) \geq NB_{A1}^{Alt} \tag{4}$$

$$\tau_{A2}\rho(\alpha_1,\alpha_2)[a_{A2}(\alpha_1,\alpha_2) - \bar{a}_{A2}] - \alpha_2 C_{A2}(a_{A2}(\alpha_1,\alpha_2)) \geq NB_{A2}^{Alt} \tag{5}$$

where $NB_j^{Alt} \geq 0, j = A1, A2$, represents the net benefit of Association j's next-best alternative. Equations (**??**) and (**??**) are therefore the associations' respective participation constraints, i.e., the necessary profitability conditions that must be met before the associations will agree to participate in the WQT market with NRCS subsidization. Letting μ_1 and μ_2 represent the multipliers on equations (**??**) and (**??**), respectively, and λ the multiplier on equation (**??**), the optimality conditions for the NRCS' problem are (**??**), (**??**), (**??**), and

$$sC_{A1}(\cdot) - [1-\alpha_1]\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] = [1-\alpha_2]\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] - \mu_1\Omega_{11} - \mu_2\Omega_{12} \tag{6}$$

$$sC_{A2}(\cdot) - [1-\alpha_2]\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] = [1-\alpha_1]\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_2}\right] - \mu_1\Omega_{21} - \mu_2\Omega_{22} \tag{7}$$

$$\mu_1 = \mu_2 \tag{8}$$

where s is a scaling factor to account for the α_js being measured as percentages rather than as physical units, e.g. dollars, and

$$\Omega_{11} = sC_{A1}(\cdot) + \alpha_1\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] - \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_1}\right][a_{A1}(\cdot) - \bar{a}_{A1}] - \rho(\cdot)\tau_{A1}\left[\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right]$$

$$\Omega_{12} = \alpha_2\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] - \tau_{A2}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_1}\right][a_{A2}(\cdot) - \bar{a}_{A2}] - \rho(\cdot)\tau_{A2}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right]$$

$$\Omega_{21} = \alpha_1\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A12}(\cdot)}{\partial \alpha_2}\right] - \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_2}\right][a_{A1}(\cdot) - \bar{a}_{A1}] - \rho(\cdot)\tau_{A1}\left[\frac{\partial a_{A1}(\cdot)}{\partial \alpha_2}\right]$$

$$\Omega_{22} = sC_{A2}(\cdot) + \alpha_2\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] - \tau_{A2}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_2}\right][a_{A2}(\cdot) - \bar{a}_{A2}] - \rho(\cdot)\tau_{A2}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right].$$

Equations (**??**) and (**??**) solve for the NRCS' optimal cost-share rates α_1 and α_2, respectively. The left-hand sides of these two equations represent the marginal benefits (in terms of reduced NRCS cost-share obligations) associated with incremental increases in the respective α_js, while the right-hand sides represent the associated marginal costs. For example, in equation (**??**) marginal benefits accrue to the NRCS from (i) a reduction in the abatement effort (and thus the corresponding abatement cost) of Association $A1$ (i.e., the term $[1-\alpha_1]\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] < 0$) and (ii) a direct reduction in the abatement cost of the association (i.e., the term $sC_{A1}(\cdot) > 0$). Marginal costs consist of (i) a "cross-effect" increase in the abatement effort (and corresponding abatement cost) of Association $A2$ (i.e., the term $[1-\alpha_2]\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] > 0$) and (ii) the added net costs associated with changes in the profitabilities of Associations $A1$ and $A2$, and thus the ability of the NRCS to satisfy participation constraints (**??**) and (**??**), i.e. the terms $\mu_1\Omega_{11}$ and $\mu_2\Omega_{12}$. Finally, equation (**??**) results from the NRCS' choice of the initial allocations $\bar{a}_j, j = A1, A2$ and reflects the fact that the NRCS values the participation of each association equally on the margin. Solving equations (**??**) – (**??**) simultaneously establishes the benchmark full-information equilibrium, characterized by the optimal value set $\left(\alpha_1^*, \alpha_2^*, a_j^*, \bar{a}_j^*, \rho^*, \mu_1^*, \mu_2^*, \lambda^*\right), j = A1, A2$, and P.

2.2 The Case of Incomplete Information

Two types of information uncertainty limit the NRCS's ability to implement the full-information solution. The first pertains to the diffuse nature of NPS emissions, and thus the ultimate effectiveness of an association's abatement effort. Abatement effort at the source does not necessarily translate into a certain measured reduction in concentration at the receptor point, due to environmental factors such as source-specific BMP effectiveness, seasonal or weather-related variations, and the general limitations of hydrological and topographical data. The second type of uncertainty pertains to the ability of an association to misrepresent itself, e.g., to "hide" information from the NRCS with respect to whether it is able to abate at high or low cost.[13]

To capture the potential effect of these two types of uncertainty we introduce parameter π_{ij}, $0 < \pi_{ij} < 1$, for Associations i and j, where i defines what an association truely is and j defines how the association actually behaves in its interaction with the NRCS (and thus subsequently in

[13]For example, if a low-cost association believes that the NRCS's cost-share rule is monotone in abatement costs (i.e., associations with higher abatement cost functions tend to receive higher subsidy rates) the association may have incentive to misrepresent itself as high cost in order to receive a lower α_j.

the WQT market).[14] For example, π_{11} represents the probability of Association $A1$'s abatement effort translating into its expected concentration reduction when it chooses to abate in accordance with its true abatement technology, rather than in accordance with some other technology, e.g., Association $A2$'s true technology.[15] Similarly, π_{12} represents the probability of Association $A1$'s abatement effort translating into its expected reduction when it chooses to abate in accordance with Association $A2$'s true technology. We henceforth assume

$$\pi_{ii} \neq \pi_{ij} \tag{9}$$

for all i and j, $i \neq j$. In other words, uncertainty varies with abatement technology (or BMP effectiveness) at a given location. One would expect this to be the case, since different abatement technologies at a given location (implemented by a given association) are likely to be associated with different probabilities of obtaining expected concentration reductions at the receptor. We further assume

$$\pi_{ii} = \pi_{ji} \tag{10}$$

for all i and j, $i \neq j$, i.e., uncertainty does not vary across location for a given abatement technology. Thus, if two different associations adopt the same abatement technology the uncertainty associated with the resulting concentration reductions is uniform; differences in concentration reduction are motivated solely by the associations' respective transfer coefficients.[16]

To motivate our analysis of incomplete information, we assume that Association $A1$'s true technology is low cost and Association $A2$'s is high cost. Further, the difference in costs abides by the *single-crossing property*, i.e.,

$$\frac{\partial C_{A2}(a)}{\partial a} > \frac{\partial C_{A1}(a)}{\partial a} \tag{11}$$

for all abatement levels a. Thus, not only is Association $A2$'s abatement cost everywhere larger than Association $A1$'s, but it also increases at a faster rate.

[14] In other words, by portraying itself as Association j, Association i must actually produce abatement according to Association j's abatement technology, which in turn fixes the abatement that the association ultimately "brings to market."

[15] By "true" we mean a fiscally feasible technology that would be the best choice for the association from an environmental perspective. By "expected" concentration reduction, we mean the reduction that is expected to occur under average environmental conditions and ideal application of the true technology at the source.

[16] The implications of relaxing this assumption are explored below.

As a result of the uncertainty inherent in each association's measured concentration reductions, the abatement efforts that the associations effectively "bring to market" are themselves uncertain. Thus, Association j's cost minimization problem in the second stage of its game with the NRCS is,[17]

$$\min_{a_j} \quad \alpha_j C_j(a_j) - \rho\tau_j[\pi_{jj}a_j - \bar{a}_j], \quad j = A1, A2$$

which results in optimality conditions

$$\alpha_j \left[\frac{\partial C_j}{\partial a_j}\right] = \pi_{jj}\rho\tau_j, \quad j = A1, A2. \tag{12}$$

The market clearing condition (**??**) under incomplete information becomes

$$\bar{A} = \pi_{11}\tau_{A1}\bar{a}_{A1} + \pi_{22}\tau_{A2}\bar{a}_{A2} + \tau_P\bar{a}_P \leq \pi_{11}\tau_{A1}a_{A1} + \pi_{22}\tau_{A2}a_{A2} + \tau_P a_P \tag{13}$$

Together with (**??**) and (**??**), (**??**) characterizes the WQT market equilibrium.

In the first stage of the game, the NRCS again chooses the associations' respective cost-share rates α_j and initial allocations $\bar{a}_j$ to minimize its cost-share obligation,

$$\min_{\alpha_j, \bar{a}_j} \quad [1-\alpha_1]C_{A1}(a_{A1}(\alpha_1,\alpha_2)) + [1-\alpha_2]C_{A2}(a_{A2}(\alpha_1,\alpha_2))$$

subject to (**??**) and,

$$\tau_{A1}\rho(\alpha_1,\alpha_2)[\pi_{11}a_{A1}(\alpha_1,\alpha_2) - \bar{a}_{A1}] - \alpha_1 C_{A1}(a_{A1}(\alpha_1,\alpha_2)) \geq NB_{A1}^{Alt} \tag{14}$$

$$\tau_{A2}\rho(\alpha_1,\alpha_2)[\pi_{22}a_{A2}(\alpha_1,\alpha_2) - \bar{a}_{A2}] - \alpha_2 C_{A2}(a_{A2}(\alpha_1,\alpha_2)) \geq NB_{A2}^{Alt} \tag{15}$$

$$\begin{aligned} &\tau_{A1}\rho(\alpha_1,\alpha_2)[\pi_{11}a_{A1}(\alpha_1,\alpha_2) - \bar{a}_{A1}] - \alpha_1 C_{A1}(a_{A1}(\alpha_1,\alpha_2)) \geq \\ &\qquad \tau_{A1}\rho(\alpha_1,\alpha_2)[\pi_{12}a_{A2}(\alpha_1,\alpha_2) - \bar{a}_{A2}] - \alpha_2 C_{A1}(a_{A2}(\alpha_1,\alpha_2)) \end{aligned} \tag{16}$$

$$\tau_{A2}\rho(\alpha_1,\alpha_2)[\pi_{22}a_{A2}(\alpha_1,\alpha_2) - \bar{a}_{A2}] - \alpha_2 C_{A2}(a_{A2}(\alpha_1,\alpha_2)) \geq$$

[17] In equilibrium, the associations do not minimize expected costs over the choices of both a_j and a_{-j} since, as a result of the NRCS's choices in the first stage, they are induced to portray themselves truthfully. Truthful revelation occurs as a result of the NRCS satisfying both participation and incentive-compatibility constraints in its first-stage problem (see below).

$$\tau_{A2}\rho(\alpha_1,\alpha_2)[\pi_{21}a_{A1}(\alpha_1,\alpha_2)-\bar{a}_{A1}]-\alpha_1 C_{A2}(a_{A1}(\alpha_1,\alpha_2)) \quad (17)$$

where we henceforth assume $NB_{A1}^{Alt} = NB_{A2}^{Alt} \geq 0$. Equations (**??**) and (**??**) are participation constraints (**??**) and (**??**), respectively, adjusted for the presence of uncertainty. Equations (**??**) and (**??**) are standard self-selection (or incentive-compatibility) constraints. For example, (**??**) states that cost shares α_j are chosen such that the profit Association $A1$ obtains from adopting its true abatement technology is never less than the profit it would obtain by adopting Association $A2$'s technology. Likewise for Association $A2$.

To begin, we compare constraints (**??**) and (**??**). Using (**??**) and the fact that Association $A1(A2)$ is low(high) cost, i.e., $C_{A2}(a) > C_{A1}(a)$ for all a, we see that when

$$\tau_{A1}\rho(\alpha_1,\alpha_2)[\pi_{12}a_{A2}(\alpha_1,\alpha_2)-\bar{a}_{A2}]-\alpha_2 C_{A1}(a_{A2}(\alpha_1,\alpha_2)) \geq$$

$$\tau_{A2}\rho(\alpha_1,\alpha_2)[\pi_{22}a_{A2}(\alpha_1,\alpha_2)-\bar{a}_{A2}]-\alpha_2 C_{A2}(a_{A2}(\alpha_1,\alpha_2)) \geq NB_{A2}^{Alt} = NB_{A1}^{Alt} \geq 0,$$

e.g., when $\tau_{A1} \geq \tau_{A2}$ (**??**) is not binding, implying that (**??**) is.[18] By contrast, when conditions are such that (**??**) does not bind, e.g., when $\tau_{A2} \geq \tau_{A1}$ by a sufficient amount, then as long as (**??**) binds (**??**) will bind as well.

We can compare constraints (**??**) and (**??**) in a similar manner. Assuming (**??**) binds, it can be substituted to (**??**) resulting in,

$$\frac{\alpha_2\tau_{A2}C_{A1}(a_{A2}(\alpha_1,\alpha_2))}{\tau_{A1}}-\frac{\alpha_1\tau_{A2}C_{A1}(a_{A1}(\alpha_1,\alpha_2))}{\tau_{A1}} \geq \alpha_2 C_{A2}(a_{A2}(\alpha_1,\alpha_2))-\alpha_1 C_{A2}(a_{A1}(\alpha_1,\alpha_2))$$

By (**??**), $a_{A1}(\cdot) > a_{A2}(\cdot)$ (which is a natural outcome given $C_{A2}(a) > C_{A1}(a)$ for all a), and small enough differences between α_1 and α_2, on the one hand, and between τ_{A1} and τ_{A2} on the other, (**??**) does not bind (implying that (**??**) binds). This particular result – where (**??**) and (**??**) are the binding constraints – is common in the hidden-information literature (e.g., Varian, 1992; Mas-Colell, et al., 1995). It results in the NRCS's first-stage optimality conditions, (**??**), binding (**??**)

[18] $\tau_{A1} \geq \tau_{A2}$ is sufficient but not necessary for the above inequality to hold.

and (**??**), and

$$sC_{A1}(\cdot) - [1-\alpha_1]\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] = [1-\alpha_2]\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] - \mu_1\Psi_{11} - \mu_2\Psi_{12} \tag{18}$$

$$sC_{A2}(\cdot) - [1-\alpha_2]\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] = [1-\alpha_1]\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_2}\right] - \mu_1\Psi_{21} - \mu_2\Psi_{22} \tag{19}$$

$$-p\mu_2 = \pi_{11}\lambda \tag{20}$$

$$p(\tau_{A2}\mu_1 - \tau_{A1}\mu_2) = -\pi_{22}\tau_{A2}\lambda \tag{21}$$

where μ_1 and μ_2 now represent the multipliers on constraints (**??**) and (**??**), respectively, λ is the multiplier on (**??**), and

$$\Psi_{11} = -\alpha_2\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] + \tau_{A2}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_1}\right][\pi_{22}a_{A2}(\cdot) - \bar{a}_{A2}] + \rho(\cdot)\tau_{A2}\pi_{22}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right]$$

$$\begin{aligned}\Psi_{12} = {} & -sC_{A1}(\cdot) - \alpha_1\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] - \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_1}\right][\pi_{12}a_{A2}(\cdot) - \bar{a}_{A2}] + \rho(\cdot)\tau_{A1}\pi_{11}\left[\frac{\partial a_{A1}(\cdot)}{\partial \alpha_1}\right] \\ & + \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_1}\right][\pi_{11}a_{A1}(\cdot) - \bar{a}_{A1}] - \rho(\cdot)\tau_{A1}\pi_{12}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right] + \alpha_2\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_1}\right]\end{aligned}$$

$$\Psi_{21} = -\alpha_2\left[\frac{\partial C_{A2}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] + \tau_{A2}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_2}\right][\pi_{22}a_{A2}(\cdot) - \bar{a}_{A2}] + \rho(\cdot)\tau_{A2}\pi_{22}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] - sC_{A2}(\cdot)$$

$$\begin{aligned}\Psi_{22} = {} & sC_{A1}(\cdot) - \alpha_1\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A1}(\cdot)}\frac{\partial a_{A1}(\cdot)}{\partial \alpha_2}\right] - \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_2}\right][\pi_{12}a_{A2}(\cdot) - \bar{a}_{A2}] + \rho(\cdot)\tau_{A1}\pi_{11}\left[\frac{\partial a_{A1}(\cdot)}{\partial \alpha_2}\right] \\ & + \tau_{A1}\left[\frac{\partial \rho(\cdot)}{\partial \alpha_2}\right][\pi_{11}a_{A1}(\cdot) - \bar{a}_{A1}] - \rho(\cdot)\tau_{A1}\pi_{12}\left[\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right] + \alpha_2\left[\frac{\partial C_{A1}(\cdot)}{\partial a_{A2}(\cdot)}\frac{\partial a_{A2}(\cdot)}{\partial \alpha_2}\right]\end{aligned}$$

Solving equations (**??**) and (**??**) – (**??**) simultaneously establishes our incomplete-information equilibrium, characterized by the value set $\left(\alpha_1', \alpha_2', a_j', \bar{a}_j', \rho', \mu_1', \mu_2', \lambda'\right), j = A1, A2$, and P. Similar to Varian (1992), this particular solution results in the high-cost association $A2$ receiving a cost-share from the NRCS that just makes it indifferent to participating in the WQT market. However, the low-cost association $A1$ receives a surplus that just discourages it from pretending to be a high-cost association. Further, Association $A1$ chooses to follow the full-information benchmark optimality condition for its choice of abatement effort.

To see this particular result more clearly, we now turn to numerical simulations of the basic model. As shown in the next section, these simulations are based on simple functional forms for the associations' and point source's abatement cost functions.

3 Numerical Simulations

In order to simulate possible equilibrium outcomes under full and incomplete information, we assume the following abatement cost functions for Associations $A1$ and $A2$ and point source P, respectively,[19]

$$C_{A1}(a_{A1}) = \frac{a_{A1}^2}{2}$$

$$C_{A2}(a_{A2}) = \frac{2a_{A2}^2}{3}$$

$$C_P(a_P) = a_P^2$$

Thus, Association $A2(A1)$ is high(low) cost and condition (**??**) is duly satisfied. Table 1 contains the various parameter values for our initial and alternative simulation exercises. Note that for the initial simulation we assume that Association $A1$ is "downstream" from Association $A2$ vis-a-vis the receptor point, i.e., $\tau_{A1} = 1 > \tau_{A2} = 0.95$, and that the probability of either association choosing the low-cost abatement technology is slightly higher than the probability of choosing the high-cost technology, i.e., $\pi_{11} = \pi_{21} = 0.9 > \pi_{12} = \pi_{22} = 0.8$.[20] Results for the full-information and incomplete-information scenarios for this initial simulation are presented in Table 2.

For the full-information benchmark, note that Association $A2$ receives a slightly lower subsidy in percentage terms, i.e., it shares 51% of its abatement cost while Association $A1$ shares only 49% of its cost. In contrast, $A2$ receives a lower initial abatement allocation (24.42 vs. 36.93 units). As a result of WQT, $A1$ abates more than $A2$ (79.00 vs. 54.26 units). The excess of abatement over initial allocation for each respective association is then sold to point source P, so that P can effectively meet its abatement allocation, i.e., (89.87 - 19.46) = (79.00 - 36.93) + (54.26 - 24.42). The NRCS effectively ensures that each association's surplus (i.e., profit) from participating in WQT equals the value of its next best alternative. However, point source P obtains negative profit

[19] GAMS IDE version 2.0.20.0 is used to generate the simulations. Copies of the input coding are available upon request from the authors.

[20] The values for $\tau_j, j = A1, A2$ are consistent with the solution provided in Section 2. Because the associations are comprised of possibly large numbers of NPSs and, as a result, are located in close proximity to each other in the watershed, the divergence in transfer coefficients is unlikely to be large.

(driven by the fact that it is given a relatively large initial abatement allocation and it does not receive a subsidy from the NRCS). The total cost of abatement (which is shared by the NRCS and the associations based on the cost-share ratios α_1 and α_2) amounts to approximately $2,500.

For the case of incomplete information (which is motivated by the introduction of the π_{ii} and π_{ij} terms in Table 1), some rather pronounced changes occur in the equilibrium outcome, as depicted in Table 2. In reacting to the uncertainty associated with measured abatement and association behavior, the NRCS increases its subsidization of both associations and provides a larger increase to Association $A2$ ($A1$'s cost-share falls from 49% to 45% and $A2$'s falls from 51% to 40%). In addition to this increase in subsidization of the associations, the NRCS shifts more of the initial abatement allocation toward point source P (from 89.87 units under full information to 105.30 units under incomplete information). As a result, the initial allocations to $A1$ and $A2$ are reduced (from 36.93 to 30.43 for $A1$ and from 24.42 to 22.79 for $A2$).[21] Further, more trading is induced between the associations, on the one hand, and the point source on the other. Both associations now abate more (and thus have more credits to sell to the point source), while the point source in turn abates slightly more itself. In concert with a higher equilibrium permit price, the negative profit incurred by the point source is now larger (from approximately -$3,100 to -$4,100). Although $A2$'s profit is constrained to equal the value of its next best alternative, $A1$, being the low-cost association, is able to extract a rent from the NRCS worth approximately $325. Finally, the total cost of abatement rises substantially, from approximately $2,500 to $3,800, indicating an "informational cost" of approximately $1,300, or 52%.

As indicated in Table 1, we also examine an alternative scenario where the associations' respective transfer coefficients are reversed, i.e., Association $A2$ is now located downstream from Association $A1$. We similarly reverse the uncertainty parameters to reflect a higher probability of the association choosing the high-cost abatement technology rather than the low-cost technology, i.e., $\pi_{12} = \pi_{22} = 0.9 > \pi_{11} = \pi_{21} = 0.8$. Table 3 reports the results.[22]

[21] Thus, the NRCS in effect compensates Association $A1$ for having received a smaller decrease in its cost share with a larger decrease in its initial abatement allocation.

[22] Due to the ambiguity associated with an incomplete-information equilibrium for the case of $\tau_{A2} > \tau_{A1}$ (as discussed in Section 2), we have solved the NRCS's first-stage problem for each of the three combinations of constraints that the NRCS could conceivably face, and have then selected the solution corresponding to the lowest total abatement cost. The three constraint combinations are {(**??**),(**??**)}, {(**??**),(**??**)}, and {(**??**),(**??**)}. As demonstrated in Section 2, the constraints included in combinations {(**??**),(**??**)} and {(**??**),(**??**)} cannot hold simultaneously. In addition, the constraint combination {(**??**),(**??**)} is incompatible with the NRCS's choices of $\bar{a}_j, j = A1, A2$ (a proof of this result is available from the authors upon request).

Comparing the full-information results in Table 3 with those in Table 2, we see that the resulting equilibrium changes only slightly in response to the change in the transfer coefficients and the uncertainity parameters. In relative terms, all of the results obtained for the initial full-information equilibrium are retained in this alternative equilibrium, e.g., Association $A2$ receives a slightly lower subsidy in percentage terms than Association $A1$, as well as a lower initial abatement allocation, etc. However, with respect to the incomplete-information equilibrium (which is constrained by constraint set {(**??**),(**??**)}, the changes are more pronounced.[23] Because the probability of Association $A1$ choosing the high-cost abatement technology is now higher under the alternative scenario, its cost share falls rather dramatically, from 45% to 35% (or, alternatively stated, its subsidy rate rises from 55% to 65%). The NRCS compensates for this by slightly increasing Association $A2$'s cost share rate and by eliminating $A1$'s rent payment, which is a consequence of constraint (**??**) now holding. Finally, the fall in $A1$'s cost share results in a larger total cost of abatement (from $3,800 to $4,300).

4 Sharing the WQT Surplus Within an NPS Association

Since NRCS subsidies result in surpluses for the associations through participation in WQT, e.g., the $Profit_j, j = A1, A2$ values in Tables 2 and 3, the need to allocate the surplus among a given association's members arises. To assess the effectiveness of the Shapley value as a potential surplus sharing rule, we begin by introducing some basic concepts of cooperative games that will prove useful in the succeeding analysis. Next, we show that the Shapley value results in a core allocation of a convex surplus-sharing game. A simple example concludes this section.

Because the level of analysis of the non-cooperative cost-sharing game analyzed in the previous two sections occurs at the association level (rather than at the level of the individual NPS), we are precluded from making a tight connection between the numerical results derived in Section 3 and the cooperative surplus-sharing game analyzed in this section. However, it is important to bear in mind that the ensuing discussion about cooperative games is quite general in nature, and therefore directly applies to the surpluses generated by Associations $A1$ and $A2$ in Section 3. In particular, the example presented in Section 4.3 below is generalizable to any level of surplus that might result

[23]Concomitant with footnote 19, {(**??**),(**??**)} is the constraint set that results in the lowest total cost of regulation for the NRCS.

from the type of numerical analysis performed in Section 3.

4.1 Preliminaries of Cooperative Games

Let N be a set of $n < \infty$ NPSs, with 2^N denoting the corresponding power set of N (i.e., the set of all possible coalitions based on N, including the empty and grand coalitions), and $v : 2^N \in R$ denoting the corresponding characteristic function that allocates some outcome (e.g., coalitional surplus) among each element of 2^N (with $v(\emptyset) \equiv 0$ by convention).[24] An allocation in the grand coalition of the n NPSs (i.e., the full association) is represented by $\mathbf{x} = (x_1, ...x_n)$. The core of the game (N, v) is the set of all allocations that induces each of the n NPSs to rationally join in the formation of the grand coalition, i.e.,

$$Core(N, v) = \left(\mathbf{x} : \sum_{i \in N'} x_i \geq v(N') \ for \ all \ coalitions \ N' \in 2^N \right).$$

Provided that v generates solely positive values (i.e., surplus as opposed to cost), two common types of games are typically considered – Additive ($for\ all\ disjoint\ coalitions\ N_1, N_2 \in 2^N, v(N_1 \cup N_2) = v(N_1) + v(N_2)$) and Convex ($for\ all\ N_1, N_2 \in 2^N, v(N_1 \cup N_2) + v(N_1 \cap N_2) \geq v(N_1) + v(N_2)$). The core for additive games is a singleton, i.e., $x_i = v(\{i\})$ for each NPS i. Convex games guarantee a non-empty core that includes the Shapley value (as discussed in Section 4.2 below).

We can represent the marginal contribution of NPS i to coalition N' as $v(N' \cup \{x_i\}) - v(N')$ where $x_i \notin N'$.[25] We have as many as $2^{|N'|-1}$ such values relative to all possible coalitions in $2^{N' \backslash \{i\}}$, where $\mid N' \mid$ represents the cardinality of coalition N'. An allocation obtained by a weighted average of these marginal contributions therefore accounts for the "marginality" of this NPS. A special set of weights yielding allocations

$$x_i = \sum_{N' \in 2^{N \backslash \{i\}}} \frac{\mid N' \mid !(n- \mid N' \mid -1)!}{n!} \left[v(N' \cup \{i\}) - v(N') \right], \tag{22}$$

is known as the Shapley value (SV) of game (N, v).

Calculation of the SV is straight-forward, particularly for games with small numbers of NPSs.

[24] In the context of WQT "coalitional surplus" pertains to a coalition's profit, as depicted for the grand coalitions, or Associations $A1$ and $A2$, in Tables 2 and 3 of Section 3.

[25] In the context of WQT, the marginal contribution of NPS i to coalition N' represents the added profit that coalition N' would obtain by including NPS i's abatement credits.

For example, in the three-player game with coalitional surpluses depicted in Table 4,

$$x_i = \frac{2!0!}{3!}(9-6) + \frac{1!1!}{3!}(5-3) + \frac{1!1!}{3!}(4-2) + \frac{0!2!}{3!}(1-0) = 2.$$

Similar calculations for NPSs 2 and 3 yield allocations $x_2 = 3$ and $x_3 = 4$, respectively. Calculation of the SV is extended to a six-NPS association in Section 4.3.

Since economic analysis predominantly takes marginality into account, the SV represents a reasonable candidate for the surplus-sharing problem within an NPS association. However, it is crucial that the SV is also shown to be in the core, as this is consistent with the a priori existence of the full association. As mentioned earlier, convex games ensure that the SV is included in a non-empty core. The next section demonstrates this fact.

4.2 A Convex Extension

As mentioned in Section 2, in the case where NPSs coalesce into an association, the NRCS effectively subsidizes the aggregate abatement expenses of the association. The subsidies, in turn, enable the associations to generate a surplus, or profit, from selling abatement credits in a WQT market. To see why the SV allocation of this profit exists in the non-empty core of a subsequent convex game between the NPSs of the association, consider a partition $N_1, ..., N_m$ of the full association N, where each pair N_k and $N_{k'}$ is disjoint for $k \neq k'$, and $\cup_{k=1}^{m} N_k = N$. Assume that this partition is the finest one of such partitions for which the value a_k of each coalition N_k is known. Here, the value a_k represents the aggregate level of abatement of coalition N_k. Let $\mathbf{N} = \{N_1, ..., N_m\}$ denote the collection of these coalitions. Further, we define $\tilde{\mathbf{N}}$ as the collection of each of the subsets of N that can be obtained by arbitrary unions of the elements in $\mathbf{N}$. That is, for any $N_1 \in \tilde{\mathbf{N}}$, there exists a sub-collection of $\mathbf{N}$ whose union equals N_1.

Definition 1. *The collection* $\tilde{\mathbf{N}}$ *is an algebra if it is closed under both union and complement.*

Claim 1. $\tilde{\mathbf{N}}$ *is an algebra.*

Proof. That $\tilde{\mathbf{N}}$ is closed under union is tautological given its construction of unions of the partition $\mathbf{N}$ of N. To show that $\tilde{\mathbf{N}}$ is closed under complement, let $N_1 \in \tilde{\mathbf{N}}$ be arbitrary. Then, there exists an $\mathbf{N}' \subseteq \mathbf{N}$ such that $N_1 = \cup\{\mathbf{N}' \in \mathbf{N}\}$ by definition of $\tilde{\mathbf{N}}$. Since $\mathbf{N}$ is a partition of N, we have N_1's complement $N_m \in N \backslash N_1 = \cup\{\mathbf{N}'' \in \mathbf{N} \backslash \mathbf{N}'\}$. But, by definition of $\tilde{\mathbf{N}}$ again, we have $N \backslash N_1 \in \tilde{\mathbf{N}}$. Since N_1 was chosen arbitrarily, this shows that $\tilde{\mathbf{N}}$ is closed under complement. Hence, $\tilde{\mathbf{N}}$ is an algebra. *Q.E.D.*

As mentioned above, a game v, for a set of NPSs N, is typically defined on the power set 2^N. Clearly, $\tilde{\mathbf{N}} \subseteq 2^N$. We now construct a characteristic function $\tilde{v}$ restricted to $\tilde{\mathbf{N}}$ as follows. By our definition of $\mathbf{N}$, the value a_k of each coalition $N_k \in N$ is known. Define

$$\tilde{v}(N_k) := p_k a_k, \tag{23}$$

for each $k = 1, ..., m$, where p_k is the net price of coalition N_k's credit(s) sold in the WQT market. Based on our proof of Claim 1 and the definition of $\tilde{\mathbf{N}}$, there exists a sub-collection $\mathbf{N}' \subseteq \mathbf{N}$ such that $N_1 = \cup\{N_k \in \mathbf{N}'\}$ for each $N_1 \in \tilde{\mathbf{N}} \backslash \mathbf{N}$. Thus, we can define,

$$\tilde{v}(N_1) := \sum_{N_k \in \mathbf{N}'} p_k a_k. \tag{24}$$

It is clear that $\tilde{v}$ is additive in its domain $\tilde{\mathbf{N}}$ (see Section 4.1), which leads to the following theorem.

Theorem 1. *An additive game restricted to a finite algebra of N has an extension to a convex game on 2^N.*

By this theorem we have a convex game $v : 2^N \to R$, which is extended from the $\tilde{v} : \tilde{\mathbf{N}} \to R$ constructed above. By convexity, this extended game assures that the SV exists in a non-empty core. The extended characteristic function v, given our $\tilde{v}$, is obtained as follows. For all $N_1 \in \tilde{\mathbf{N}}$, define

$$v(N_1) := \tilde{v}(N_1) \tag{25}$$

so that the definition of $\tilde{v}$ is preserved. For each $N_1 \in 2^N \backslash \tilde{\mathbf{N}}$, define

$$v(N_1) := \sum_{N_k \in N, N_k \subseteq N_1} \tilde{v}(N_k). \tag{26}$$

The resultant characteristic equation v is exactly the extension of $\tilde{v}$ as in Theorem 1.

4.3 A Simple Example

Suppose the value of an abatement credit created jointly by NPS 1 and NPS 2 is known to be 40, however the value of a credit created by each NPS separately is not known. Similarly, NPSs 3 – 5 jointly generate a credit valued at 55. NPS 6 alone generates a credit valued at 25.[26] In this case, we have $N = \{1,2,3,4,5,6\}$, $\mathbf{N} = \{\emptyset, \{1,2\}, \{3,4,5\}, \{6\}\}$, and

$\tilde{\mathbf{N}} = \{\emptyset, \{1,2\}, \{3,4,5\}, \{6\}, \{1,2,3,4,5\}, \{1,2,6\}, \{3,4,5,6\}, N\}$. The characteristic function $\tilde{v}$ restricted to $\tilde{\mathbf{N}}$ is given by (**??**) and (**??**) as $\tilde{v}(\emptyset) \equiv 0$, $\tilde{v}(\{1,2\}) = 40$, $\tilde{v}(\{3,4,5\}) = 55$, $\tilde{v}(\{6\}) = 25$, $\tilde{v}(\{1,2,3,4,5\}) = 95$, $\tilde{v}(\{1,2,6\}) = 65$, $\tilde{v}(\{3,4,5,6\}) = 80$, and $\tilde{v}(\{N\}) = 120$.

The extended characteristic function $v : 2^N \to R$ is given by (**??**) and (**??**) as $v(N_1) = \tilde{v}(N_1)$ for all $N_1 \in \tilde{\mathbf{N}}$, $v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = v(\{5\}) = 0$, $v(\{1,3\}) = v(\{1,4\}) = v(\{1,5\}) = v(\{2,3\}) = v(\{2,4\}) = v(\{2,5\}) = 0$, $v(\{3,4\}) = v(\{3,5\}) = v(\{4,5\}) = 0$, $v(\{1,3,4\}) = v(\{1,3,5\}) = v(\{1,4,5\}) = v(\{2,3,4\}) = v(\{2,3,5\}) = v(\{2,4,5\}) = 0$, $v(\{1,6\}) = v(\{2,6\}) = v(\{3,6\}) = v(\{4,6\}) = v(\{5,6\}) = 25$, $v(\{1,3,6\}) = v(\{1,4,6\}) = v(\{1,5,6\}) = v(\{2,3,6\}) = v(\{2,4,6\}) = v(\{2,5,6\}) = 25$, $v(\{3,4,6\}) = v(\{3,5,6\}) = v(\{4,5,6\}) = 25$, $v(\{1,3,4,6\}) = v(\{1,3,5,6\}) = v(\{1,4,5,6\}) = 25$, $v(\{2,3,4,6\}) = v(\{2,3,5,6\}) = v(\{2,4,5,6\}) = 25$, $v(\{1,2,3\}) = v(\{1,2,4\}) = v(\{1,2,5\}) = 40$, $v(\{1,2,3,5\}) = v(\{1,2,4,5\}) = 40$, $v(\{1,3,4,5\}) = v(\{2,3,4,5\}) = 55$, $v(\{1,2,3,6\}) = v(\{1,2,4,6\}) = v(\{1,2,5,6\}) = 65$, $v(\{1,2,3,4,6\}) = v(\{1,2,3,5,6\}) = v(\{1,2,4,5,6\}) = 65$, and $v(\{1,3,4,5,6\}) = v(\{2,3,4,5,6\}) = 80$.

This completes the list of $2^n = 64$ combinations. Though laborious, one can check that this game is convex (see Section 4.2). The SV computed by (**??**) is $\mathbf{x} = (20, 20, \frac{55}{3}, \frac{55}{3}, \frac{55}{3}, 25)$, which is, indeed, a reasonable allocation given the setting of this example. That this allocation is in the core is easy to check.

Thus, the SV can provide core allocations of any surpluses generated by Associations $A1$ and $A2$, respectively, in a convex sharing game. This, in turn, suggests that the formation of NPS associations in order to reduce the complexity of the contracting relationship with the regulatory authority also maintains a consistency with the associations' decisions to have formed in the first place.

[26] For example, due to technological limitations the regulatory authority is unable to separately monitor pollution from NPSs 1 – 5. The best it can do is monitor jointly produced pollution from NPSs 1 and 2 and from NPSs 3 – 5, respectively.

5 Summary

This paper has examined how a regulatory authority might best subsidize the participation of associations (through a cost-sharing arrangement) in a surplus-generating economic activity, and how the agents within a given association might in turn cooperatively share the surplus. Toward this end, the subgame-perfect equilibrium concept was adopted to model the "multilateral contracting" relationship between the regulatory authority and the associations when the authority has incomplete information about both the association's abatement costs and the natural environment. In the context of the nonpoint pollution problem, we find that cost-sharing in concert with water quality trading can incentivise associations to independently abate at "constrained-efficient" allocations in the presence of uncertainty regarding their loadings. The extent to which the efficient allocation is constrained by both the asymmetric information concerning the associations' costs and uncertainty about the physical characteristics of the watershed (in terms of the spatial differences among NPSs that distinguish the potency of their respective loadings) are demonstrated analytically. In numerical analysis we find that the regulator reacts to its problem of incomplete information by increasing its cost-share ratios and, in some cases, providing additional rent to the low-cost association. It also shifts more of the required abatement to the point source, for which there is no asymmetric information problem. As a result, the overall cost of regulation rises relative to the full-information benchmark.

The Shapley value was then applied to model the surplus (i.e., profit through trading) sharing game within a given association. We have shown that for convex surplus-sharing games the Shapley Value is included in a non-empty core. This suggests that by having precommitted to the Shapley value as its sharing rule, the association has maintained a consistency between its decision to have formed in the first place and its selection of the Shapley value to allocate the surplus.

In sum, our results suggest that a cost-sharing arrangement in the presence of incomplete information about both an association's abatement costs and the natural environment can successfully be coupled with the development of a market within which the associations are able to generate surplus through trading with one another. In cases where the associations can voluntarily decide whether to enter the cost-sharing arrangement with the regulator, such as for the control nonpoint pollution, the development of such a market may be a strong enough inducement for the associations to enter the cost-sharing arrangement in the first place.

References

Bolton, P. and M. Dewatripont (2005) *Contract theory*. MIT Press, Cambridge, MA.

Breetz, H.L., K. Fisher-Vanden, H. Jacobs, and C. Schary (2005) "Trust and communication: mechanisms for increasing farmers' participation in water quality trading." *Land Economics*, 81(2), 170-190.

Caplan, A.J., R.C. Cornes, and E.C.D. Silva (2000) "Pure public goods and income redistribution in a federation with decentralized leadership and imperfect labor mobility." *Journal of Public Economics*, 77(2), 265-284.

Farrow, R.S., M.T. Schultz, P. Celikkol, and G.L. Van Houtven (2005) "Pollution trading in water quality limited areas: use of benefits assessment and cost-effective trading ratios." *Land Economics*, 81(2), 191-205.

Freeman, A.M. (2002) "Environmental policy since earth day I: what have we gained?" *Journal of Environmental Perspectives*, 16(1), 125-146.

King, D.M. (2005) "Crunch time for water quality trading." *Choices*, 20(1), 71-75.

Laffont, J-J and J. Tirole (1993) *A theory of incentives in procurement and regulation*. MIT Press, Cambridge, MA.

Mas-Colell, A., M.D. Whinston, and J.R. Green (1995) *Microeconomic theory*. Oxford University Press, New York, NY.

Moulin, H. (1988) *Axioms of cooperative decision making*, Econometric Society Monographs No. 15, Cambridge University Press, Cambridge, UK.

Moulin, H. (2004) *Fair division and collective welfare*. The MIT Press, Cambridge, MA.

National Resource Conservation Service (NRCS). Retrieved from the internet on May 21, 2006 at http://www.nrcs.usda.gov/Programs/eqip/.

Shortle, J.S. and R.D. Horan (2001) "The economics of nonpoint pollution control." *Journal of Economic Surveys*, 15(3), 255-289.

Shortle, J.S. and R.D. Horan (2005) "When two wrongs make a right: second best point-nonpoint trading ratios." *American Journal of Agricultural Economics*, 87(2), 340-352.

Tomasi, T., K. Segerson, and J. Braden (1994) "Issues in the design of incentive schemes for nonpoint source pollution control." In C. Dosi and T. Tomasi (eds.), *Nonpoint source pollution regulation: issues and analysis*. Kluwer Academic Publishers, Dordrecht, Germany.

United States Environmental Protection Agency (EPA) (2005) *National water quality report*. EPA, Washington, D.C. Retrieved from the internet on June 14, 2006 at http://www.oaspb.epa.gov/waters/national_ rept.control# IMP_ STATE.

Varian, H.R. (1992) *Microeconomic analysis*, Third Edition. W.W. Norton and Company, NY.

Vossler, C.A., G.L. Poe, W.D. Schulze, and K. Segerson (2006) "Communication and incentive mechanisms based on group performance: an experimental study of nonpoint pollution control." *Economic Inquiry*, 44(4), 599-613.

Wellisch, D. (1994) "Interregional spillovers in the presence of perfect and imperfect household mobility." *Journal of Public Economics*, 55, 167-184.

Table 1: Parameter Values for Simulations of Full- and Incomplete-Information Scenarios.

Parameter	Initial Value	Alternative Value
$\bar{A}$	150	150
τ_{A1}	1	0.95
τ_{A2}	0.95	1
τ_P	1	1
$NB_{A1}^{Alt} = NB_{A2}^{Alt}$	100	100
$\pi_{11} = \pi_{21}$	0.9	0.8
$\pi_{12} = \pi_{22}$	0.8	0.9
s	0.01	0.01

Table 2: Results for Initial Simulation of Full- and Incomplete-Information Scenarios.

Variable	Full-Info Value	Incomplete-Info Value
α_1	0.49	0.45
α_2	0.51	0.40
$\bar{a}_{A1}$	36.93	30.43
$\bar{a}_{A2}$	24.42	22.79
$\bar{a}_P$	89.87	105.30
a_{A1}	79.00	89.00
a_{A2}	54.26	62.94
a_P	19.46	22.07
ρ	38.91	44.13
$Profit_{A1}$	100	424.69
$Profit_{A2}$	100	100
$Profit_P$	-3,118.20	-4,159.82
μ_1	0.35	0.68
μ_2	0.35	0.35
λ	13.73	17.16
Total Cost to Associations ($C_{A1}(\cdot) + C_{A2}(\cdot)$)	2,543.43	3,778.66

Table 3: Results for Alternative Simulation of Full- and Incomplete-Information Scenarios.

Variable	Full-Info Value	Incomplete-Info Value
α_1	0.49	0.35
α_2	0.51	0.43
$\bar{a}_{A1}$	35.65	34.07
$\bar{a}_{A2}$	26.21	27.32
$\bar{a}_P$	89.93	99.52
a_{A1}	76.64	91.38
a_{A2}	57.49	65.94
a_P	19.70	21.21
ρ	39.40	42.42
$Profit_{A1}$	100	100
$Profit_{A2}$	100	100
$Profit_P$	-3,155.10	-3,771.90
μ_1	0.39	0.19
μ_2	0.39	0.22
λ	15.46	10.22
Total Cost to Associations ($C_{A1}(\cdot) + C_{A2}(\cdot)$)	2,573.37	4,341.75

Table 4: Coalitional Surpluses for Calculating the Shapley Value.

Coalition	Coalitional Surplus
$\{1\}$	1
$\{2\}$	2
$\{3\}$	3
$\{1,2\}$	4
$\{1,3\}$	5
$\{2,3\}$	6
$\{1,2,3\}$	9