Matching Traders in a Pollution Market: The Case of Cub River, Utah*

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Abstract

This paper applies two recently developed trading algorithms to a water quality trading (WQT) market located in the Cub River sub-basin of Utah; a market that includes both point and nonpoint sources. The algorithms account for three complications that naturally arise in WQT markets: (1) combinatorial matching of traders, (2) trader heterogeneity, and (3) discreteness in abatement technology. The algorithms enable a full characterization of the market's performance by distinguishing a specific pattern of trade among market participants, which in turn results in as detailed a reduced-cost trading benchmark as possible for the basin. Contrary to the commonly held belief that relatively high point-source abatement costs necessitate nonpoint-source abatement effort, we find that in a WQT market where each source is required to reduce its pollution loadings it may be cheaper for point sources to sell abatement credits to nonpoint sources.

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1 Introduction

The efficacies of pollution permit markets are typically evaluated along one of two lines - the degree of cost efficiency obtained through trading or the extent of market liquidity, as reflected in trading volume and the time path of prices (Burtraw [3]). While the market-liquidity approach is restricted to assessing the past performance of existing markets, the cost-efficiency approach is perhaps best used to establish benchmarks for future performance, e.g., by determining reduced-cost allocations of the pollutant. Recently, Sasaki and Caplan [17] have developed two companion trading algorithms that can be used to establish detailed reduced-cost benchmarks for quantity-regulated markets (such as pollution trading) by distinguishing a specific pattern of trade among market participants.[1] The first of the companion algorithms, which Sasaki and Caplan label the "advancement algorithm," sequentially isolates least-cost trades starting from a pairing of the lowest- and highest-cost producers and then advancing forward to the next-least-cost pairing, accounting for each producer's average abatement cost and set of pairwise trading ratios at each step. The second algorithm, the "retreat algorithm," then corrects for any surplus abatement by adjusting from the final pairing backward until all surpluses are eliminated (or reduced by as much as is technically feasible).

This paper applies the advancement and retreat algorithms to a water quality trading (WQT) market located in the Cub River sub-basin of Utah. The Cub River is well-suited for this kind of application due to the availabil-

[1]Dales [6] and Montgomery [14] provide seminal discussions on the cost efficiencies associated with pollution trading. We say "reduced-cost" rather than least-cost because the algorithms result in a least-cost benchmark only under certain conditions. See the appendix in Sasaki and Caplan for details.

ity of both nutrient-loading and delivery-ratio estimates for the sub-basin's plethora of nonpoint sources.[2] By accounting for all possible point-to-point, point-to-nonpoint, and nonpoint-to-nonpoint source trades, the algorithms are able to fully characterize the market's performance. Contrary to the commonly held belief that relatively high point-source abatement costs necessitate nonpoint-source abatement effort (c.f., Freeman [7], King [10], and King and Kuch [11]), we find that in a WQT market where each source is required to reduce its pollution loadings it may be cheaper for point sources to sell abatement credits to nonpoint sources.

By delineating a specific pattern of trade that reduces aggregate abatement cost, the advancement and retreat algorithms differ markedly from the traditional simulation (e.g., linear programming) approach used in previous empirical studies. For example, McGartland and Oates [13] simulate a least-cost benchmark for emissions in the Baltimore Air Quality Control Region based on estimates of integer-step abatement cost functions for over 400 polluters.[3] While their simulation approach can be used to compare source-specific emissions under the command-and-control (CAC) and least-cost regimes, the specific pattern of trade underscoring the comparison (which would conceivably result through trading) is not discernable. Therefore, policy makers have no way of comparing actual trades with the cost-efficient trades that should occur in an optimal trading outcome. Fur-

[2]For detailed information about these estimates see Neilson, et al. [15] and Caplan, et al. [4].

[3]Atkinson and Lewis [1] and Atkinson and Tietenberg [2] use a similar approach for the St. Louis Air Quality Control Region, as does Seskin et al. [18] for the Chicago Air Quality Control Region. See O'Ryan [16] for application of this approach to air quality in Santiago, Chile. Alternatively, Kling [12] uses regression analysis to estimate abatement cost functions for the control of hydrocarbon emissions from light-duty vehicles in California. More recent examples of this general approach include Keohane [9] and Shadbegian et al. [19].

thermore, McGartland and Oates' comparison of the CAC and least-cost outcomes is based on estimated marginal cost functions for each source (that are derived from the sources' actual discrete production costs), rather than directly on the discrete costs themselves. The algorithms applied in this paper overcome these limitations.

According to Sasaki and Caplan, similar algorithms are used in logistics optimization, where solutions to scheduling and routing problems have been developed on the basis of the heuristics for the traveling salesman problem, which in turn have been developed mainly for solving problems in manufacturing, such as the printed-circuit-board-drilling problem (IBM [8]). For example, IBM [8] has recently developed a vehicle routing planner (VRP) that finds optimal routes for delivery trucks on a given road network, where "optimal" in this sense means the minimum number of trucks required under capacity, time, and customer-demand constraints. Rather than delineating the specific pattern of trade among a group of traders in the Cub River sub-basin, as our algorithms do in this paper, the VRP algorithm delineates an ordinal routing pattern among a set of truck depots. Conceptually speaking, the companion trading algorithms and the VRP algorithms are quite similar.

The next section presents Sasaki and Caplan's least-cost WQT model. Section 3 then describes how the companion algorithms work, first by reiterating Sasaki and Caplan's basic intuition and then by building upon their simple two-trader example. Section 4 provides a brief description of the Cub River sub-basin, and Section 5 describes the sub-basin's WQT data and discusses the results obtained from applying the companion trading algorithms to the data. Section 6 concludes.

2 A Least-Cost Water Quality Trading Model

Consider a market for trading abatement credits among $i = \{1, 2, \ldots, n\}$ traders (a trader is referred to synonymously as a producer, a polluter, a seller, or a purchaser, depending upon the context). For future reference, the set of these traders is denoted by N. Further, let $R(i)$ represent the required abatement level for producer i set by regulation.

Each producer has his own abatement capability and associated cost structure related to a countable number of technology options, or steps. For example, a given producer's first-step technology might produce 100 units of abatement followed by 40 units in the second step, and so on.[4] In this regard, let $S(i)$ represent the number of technology steps implemented by producer i (which for future reference defines the function $S : N \to \mathbb{N} \cup \{0\}$); let $A(i, k)$ represent the abatement level achieved by producer i in his k^{th} technology step; and let $C(i, k)$ represent the corresponding abatement cost incurred by producer i in the k^{th} technology step.[5]

With the above definitions, total abatement and associated total abatement cost incurred by producer i are given by $\sum_{k=1}^{S(i)} A(i, k)$ and $\sum_{k=1}^{S(i)} C(i, k)$, respectively. In the absence of trading, the authority's regulations are met only when $R(i) \leqslant \sum_{k=1}^{S(i)} A(i, k)$ for all i. However, trading enables producer i to sell any extra abatement, i.e., abatement credits, $\sum_{k=1}^{S(i)} A(i, k) - R(i) > 0$, to other producers (who perhaps experience higher abatement costs). In

[4] What distinguishes specific technology steps is not the passage of time per se, but rather the inherent characteristics of the technology itself. Thus, in what follows we can safely eschew the need to discount the costs associated with successive technology steps.

[5] See Caplan [5] for a detailed discussion of the implications of discrete abatement.

this regard, let $P(i,j)$ represent the abatement credits that trader i sells to trader j, which in turn defines the function $P: N \times N \to \mathbb{R}_+$.

Heterogeneity (e.g., due to geographic spatiality) among producers complicates trading. For example, abatement of 100 kilograms by "upstream" polluter 1 may be equivalent to only abatement of 80 kilograms by "downstream" polluter 2 in terms of a given receptor point. As a result, 80 kilograms of abatement by polluter 2 could account for 100 kilograms of abatement for polluter 1 if abatement credits are exchangeable. Hence, all else equal, it is preferable for "receptor-sensitive" polluters such as polluter 2 to abate excessively and sell their credits to "receptor-insensitive" polluters such as polluter 1.

As in Sasaki and Caplan, we assume that trading ratios can be represented by a matrix T, where $T(i,j)$ is the trading ratio between seller i and purchaser j.[6] Given T, $P(i,j)/T(i,j)$ equals the effective abatement credits that trader i sells to trader j. If $T(i,j) < 1$ (respectively, > 1), then the quantity "swells" (respectively, "shrinks") when trader j purchases credits from trader i.

For example, suppose polluter 1 is located upstream from polluter 2, and thus, due to spatial differences, 20 percent of pollutants from polluter 1 and 25 percent of pollutants from polluter 2 transmit to the receptor. In this case, abatement of 100 kilograms by polluter 1 translates to 20 kilograms at the receptor, which, in turn, is equivalent to abatement of 80 kilograms by polluter 2. Relatively speaking then, polluter 2's trading ratio with polluter 1 is $T(2,1) = 80/100$ and polluter 1's trading ratio with polluter 2

[6] Trading ratios are themselves ratios of the purchaser's and seller's respective delivery ratios.

is $T(1,2) = 100/80$. The trading ratio matrix for this example is therefore

$$T = \begin{pmatrix} 1 & 5/4 \\ 4/5 & 1 \end{pmatrix}.$$

Suppose further that the two polluters have the same constant marginal cost of abatement and that both polluters are required to abate $R(i) = 100$ kilograms each. Then, the optimal (i.e., least-cost) solution is that polluter 2 abates 180 kilograms and sells her credit of 80 kilograms to polluter 1, i.e., $P(2,1) = 80$. This fulfills the abatement requirement for polluter 1 as well since $P(2,1)/T(2,1) = 100 = R(1)$. In this case, polluter 1's abatement credits "swell" from 80 to 100 by trading with polluter 2.

As shown in Sasaki and Caplan, the regulator's problem is to minimize total abatement cost among all producers. This problem is constrained by the achievement of a required abatement level by each producer, including net traded credits. Letting $\mathcal{S}$ and $\mathcal{P}$ denote the sets of all possible functions $S : N \to \mathbb{N} \cup \{0\}$ and $P : N \times N \to \mathbb{R}_+$, respectively, the regulator's problem is therefore expressed as

$$\min_{P\in\mathcal{P}, S\in\mathcal{S}} \quad \sum_{i=1}^{n} \sum_{k=1}^{S(i)} C(i,k) \tag{1}$$

$$\text{subject to} \quad \sum_{k=1}^{S(i)} A(i,k) + \sum_{j=1}^{n} P(j,i)/T(j,i) - \sum_{j=1}^{n} P(i,j) \geqslant R(i) \text{ for each } i \in N \tag{2}$$

$$\text{and} \quad P(i,j) > 0 \Rightarrow P(j,i) = 0 \text{ for each pair } i,j \in N. \tag{3}$$

The left-hand side of (2) is producer i's own-abatement level plus effective credits purchased minus credits sold. Equation (3) prohibits bidirectional trading. Also, (3) implies that the diagonal of P is zero.

For convenience define the functional $F : N \times \mathcal{S} \times \mathcal{P} \to \mathbb{R}$ by

$$F(i,S,P) = R(i) + \sum_{j=1}^{n} P(i,j) - \sum_{k=1}^{S(i)} A(i,k) - \sum_{j=1}^{n} P(j,i)/T(j,i).$$

Constraint (2) then reduces to

$$F(i, S, P) \leqslant 0 \text{ for each } i \in N. \tag{4}$$

When $F(i, S, P)$ is positive, producer i incurs an abatement deficit. Constraint (4) is thus equivalent to the elimination of an abatement deficit for each producer.

3 Basic Intuition and a Two-Trader Example

3.1 The Basic Intuition

Begin, in iteration $m = 0$, with an initial state of no abatement (i.e., $S_0(i) = 0$ for all $i \in N$) and thus no trading (i.e., $P_0(i, j) = 0$ for all $i, j \in N$). The advancement algorithm iteratively updates S_m and P_m to S_{m+1} and P_{m+1}, respectively, such that exactly one producer "advances" her abatement step in each iteration, aiming to reduce abatement deficits $F(i, S_m, P_m)$. In other words, in the $(m+1)$-st iteration, an increment occurs as $S_{m+1}(i) = S_m(i) + 1$ for exactly one $i \in N$. Producer i's resultant abatement may then be distributed to other producers through trading, and P is accordingly modified. The process is iterated so that S and P eventually fulfill constraint (4) while keeping the associated total cost (1) as small as possible. The algorithm ends at the M-th iteration where constraint (4) is first satisfied.

For each iteration, say, in the $(m+1)$-st iteration, the primary task is to determine for which producer i should the abatement step $S_m(i)$ be advanced. If producer i advances, the resultant abatement level and associated cost are $A(i, S_m(i) + 1)$ and $C(i, S_m(i) + 1)$, respectively. It may be intuitively natural to advance the step of the trader i who has the least average abatement cost $C(i, S_m(i)+1)/A(i, S_m(i)+1)$. However, this idea is cursory; the denominator (or the quantity) may be further swelled (respectively, shrunk) via trading by the trading ratios, thus reducing (respectively, increasing) effective average abatement cost. Recall from Section 2 that the quantity of producer i's abatement swells greater as the trading ratio $T(i, j)$ is smaller. Thus, seller i should preferably assign the highest precedence of selling her abatement to prospective purchaser j who has the lowest trading ratio $T(i, j)$. As such, we need to queue the prospective purchasers of i's abatement in the order of increasing trading ratios, which SC call a "rearrangement."

Since by the m-th iteration those who have fulfilled the constraint, $F(j, S_m, P_m) \leqslant 0$, are not included in the list of prospective purchasers of seller i' abatement, $\tilde{N}_m = \{j \in N \mid F(j, S_m, P_m) > 0\}$ is considered as the list of prospective purchasers. $\tilde{N}_m$ can include seller i herself, but the quantity that she sells to herself is necessarily reinterpreted as unsold production. Suppose that $j_1, j_2, \ldots$ is the rearranged list of the potential purchasers $\tilde{N}_m$. Hence, trader i assigns the highest precedence of selling her abatement to purchaser j_1, followed by j_2, and so on. There are three cases to consider in calculating trader i's effective average abatement cost:

CASE 1: If trader i's next-step abatement $A(i, S_m(i)+1)$ is not enough to meet purchaser j_1's abatement deficit (i.e., $A(i, S_m(i)+1)/T(i, j_1) < F(j_1, S_m, P_m)$), then the resultant swelled quantity is $A(i, S_m(i)+1)/T(i, j_1)$. Hence, trader i's effective average abatement cost is

$$\frac{C(i, S_m(i)+1)}{A(i, S_m(i)+1)/T(i, j_1)}. \tag{5}$$

CASE 2: If trader i, in her next step, is able to abate an amount sufficient to fulfill the sum of *all* prospective purchasers' abatement deficits, then the swelled/shrunk quantity as a result of trading is $\sum_{j \in \tilde{N}_m} F(j, S_m, P_m)$. (Note that $F(j, S_m, P_m)$ is already in purchaser j's quantity scale, thus there is no need to divide it by $T(i, j)$.) If we denote the number of purchasers by $\tilde{n} = |\tilde{N}_m|$, then this swelled/shrunk quantity is rewritten as $\sum_{\alpha=1}^{\tilde{n}} F(j_\alpha, S_m, P_m)$, and trader i's effective average abatement cost is

$$\frac{C(i, S_m(i) + 1)}{\sum_{\alpha=1}^{\tilde{n}} F(j_\alpha, S_m, P_m)}. \tag{6}$$

CASE 3: If the situation is neither CASE 1 nor CASE 2, then trader i is able to fulfill at least one prospective purchaser j_1's abatement deficit, but cannot fulfill that of everyone. Thus, let $\alpha^*(1 \leqslant \alpha^* < \tilde{n})$ be the number of purchasers (possibly including herself) whose deficits can be fulfilled as a result of trader i's selling her next-step abatement. Then, the fulfillment amounts to the swelled/shrunk quantity $\sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m)$. Yet, this swelled/shrunk quantity is not all the benefits of trader i's abatement. She may still have remnant $A(i, S_m(i) + 1) - \sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m) T(i, j_\alpha)$ even after fulfilling α^* purchasers' deficits. Even though she cannot completely fulfill the deficit of the $(\alpha^* + 1)$-st purchaser (by definition of α^*), she can still sell this remnant to prospective purchaser j_{α^*+1}, the swelled/shrunk quantity of which is

$$\frac{A(i, S_m(i) + 1) - \sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m) T(i, j_\alpha)}{T(i, j_{\alpha^*+1})}.$$

In total, the swelled/shrunk quantity as a result of trader i's abatement is

$$\Omega := \sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m) + \frac{A(i, S_m(i) + 1) - \sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m) T(i, j_\alpha)}{T(i, j_{\alpha^*+1})},$$

and trader i's effective average abatement cost is

$$C(i, S_m(i) + 1)/\Omega \tag{7}$$

Recall that the purpose of calculating these effective average abatement costs was to select a trader who, in the next step, has the least effective average cost. The algorithm lets this trader i^* advance her step, thus $S_{m+1}(i^*) = S_m(i^*) + 1$ whereas $S_{m+1}(i) = S_m(i)$ for all $i \neq i^*$. The abatement level $A(i^*, S_{m+1}(i^*))$ will be distributed (i.e., sold) to successive traders in seller i^*'s rearrangement, $j_1, j_2, \ldots, j_{\tilde{n}}$. If seller i^* can fulfill trader j_α's current abatement deficit, then the net amount $P(i^*, j_\alpha)$ sold by trader i^* to trader j_α increments by $F(j_\alpha, S_m, P_m)T(i^*, j_\alpha)$. If CASE 3 holds, the remnant sold by trader i^* to the $(\alpha^* + 1)$-st purchaser is $A(i^*, S_{m+1}(i^*)) - \sum_{\alpha=1}^{\alpha^*} F(j_\alpha, S_m, P_m)T(i^*, j_\alpha)$, by which $P(i^*, j_{\alpha^*+1})$ increments.

In the course of repeated iterations, it is possible that a trader who acted as a net seller in earlier iterations of the algorithm turns out to be a net purchaser in later iterations (or vice-versa). This can occur, for example, if a trader in her first step has a low effective average abatement cost but incomparably high effective average cost in her second step. To comply with (3), the algorithm at the end of each iteration modifies P so that only net sellers have positive values.

Further, as Sasaki and Caplan show, the advancement algorithm can fall short of the strict optimality associated with the regulator's problem in Section 2 due to redundancy in the producers' abatement steps. This possibility is best shown by force of example.

3.2 An Example

The first part of this example provides a case where the advancement algorithm alone solves the cost-minimization problem presented in Section 2. The second part of the example then demonstrates how the advancement algorithm alone can fail to to solve the cost-minimization problem, and how the retreat algorithm can be used to further reduce total abatement cost.[7]

Consider a two-polluter community $N = \{1, 2\}$ where both polluters are supposed to abate $R(1) = R(2) = 100$ kilograms. As in Section 2, let the trading ratios be $T(1, 2) = 5/4$ and $T(2, 1) = 4/5$. Suppose polluter 1 has only one abatement step whose attributes are $A(1, 1) = 100$ and $C(1, 1) = 100$, and polluter 2 has two abatement steps whose attributes are $A(2, 1) = 90$, $C(2, 1) = 90$, $A(2, 2) = 90$, and $C(2, 2) = 100$.

ITERATION 1: Currently, $S_0(1) = S_0(2) = 0$, $P_0(1, 2) = P_0(2, 1) = 0$, and $F(1, S_0, P_0) = F(2, S_0, P_0) = 100$. First, for the possible seller $i = 1$, the rearranged list of possible purchasers is $\{1, 2\}$ and $\alpha^* = 1$. By (5) – (7), trader 1's effective average abatement cost is

$$\begin{aligned} EAC(1, S_0(1) + 1) &= \frac{C(1, S_0(1) + 1)}{F(1, S_0, P_0) + \frac{A(1, S_0(1)+1) - F(1, S_0, P_0)T(1,1)}{T(1,2)}} \\ &= \frac{100}{100 + \frac{100 - 100 \cdot 1}{5/4}} = 1 \end{aligned}$$

Second, for the possible seller $i = 2$, the rearranged list of possible purchasers is also $\{1, 2\}$ and $\alpha^* = 1$. Again by (5) – (7), trader 2's effective average

[7] Sasaki and Caplan discuss the general conditions under which the advancement and retreat algorithms together solve the cost-minimization problem.

abatement cost is

$$\begin{aligned} EAC(2, S_0(2)+1) &= \frac{C(2, S_0(1)+1)}{F(1, S_0, P_0) + \frac{A(2,S_0(2)+1) - F(1,S_0,P_0)T(2,1)}{T(2,2)}} \\ &= \frac{90}{100 + \frac{90-100\cdot(4/5)}{1}} = \frac{9}{11}. \end{aligned}$$

Since $EAC(1, S_0(1)+1) > EAC(2, S_0(2)+1)$, we choose $i^* = 2$ and advance her step as $S_1(2) = S_0(2)+1 = 1$. But we retain $S_1(1) = S_0(1) = 0$. Since $\alpha^* = 1$ and the rearranged list of purchasers is $\{1, 2\}$ for seller $i^* = 2$, trading fulfills purchaser 1's abatement deficit. For this, the seller $i^* = 2$ sells $F(1, S_0, P_0)T(2, 1) = 100 \cdot (4/5) = 80$ to purchaser 1, and sells its remnant $A(2, S_1(2)) - F(1, S_0, P_0)T(2, 1) = 90 - 80 = 10$ to herself.

ITERATION 2: Currently, $S_1(1) = 0$, $S_1(2) = 1$, $P_1(1, 2) = 0$, $P_1(2, 1) = 80$, $F(1, S_1, P_1) = 0$, and $F(2, S_1, P_1) = 90$. First, for the possible seller $i = 1$, the rearranged list of possible purchasers is $\{2\}$ and $\alpha^* = 0$. Trader 1's effective average cost is

$$EAC(1, S_1(1)+1) = \frac{C(1, S_1(1)+1)}{A(1, S_1(1)+1)/T(1, 2)} = \frac{100}{100/(5/4)} = \frac{5}{4}.$$

Second, for the possible seller $i = 2$, the rearranged list of possible purchasers is also $\{2\}$ and $\alpha^* = 1$. Trader 2's effective average cost is

$$EAC(2, S_1(2)+1) = \frac{C(2, S_1(2)+1)}{F(2, S_1, P_1)} = \frac{100}{90} = \frac{10}{9}.$$

Since $EAC(1, S_1(1)+1) > EAC(2, S_1(2)+1)$, we choose $i^* = 2$ again, and advance her steps as $S_2(2) = S_1(2) + 1 = 2$. But we retain $S_2(1) = S_1(1) = 0$. Since $\alpha^* = 1$ and the rearranged list of purchasers is $\{2\}$ for the seller $i^* = 2$, trading fulfills the purchaser 2's own abatement deficit. For this, the seller $i^* = 2$ sells $F(2, S_1, P_1)T(2, 2) = 90 \cdot 1 = 90$ to herself. Currently, $S_2(1) = 0$, $S_2(2) = 2$, $P_2(1, 2) = 0$, $P_2(2, 1) = 80$, $F(1, S_2, P_2) =$

0, and $F(2, S_2, P_2) = 0$. Since constraint (4) is satisfied, we have terminated the iterations.

The final policy is, therefore, $S(1) = 0$, $S(2) = 2$, $P(1,2) = 0$, and $P(2,1) = 80$. In other words, polluter 1 abates nothing while polluter 2 abates $\sum_{k=1}^{S(2)} A(2,k) = 180$ kilograms, out of which $P(2,1) = 80$ kilograms are sold to polluter 1, which actually amounts to $P(2,1)/T(2,1) = 100$ kilograms of swelled quantity equaling the required abatement $R(1) = 100$ for polluter 1. Comparing this result with the example presented in Section 2, the resultant total cost is $\sum_{i=1}^{2} \sum_{k=1}^{S(i)} C(i,k) = C(2,1) + C(2,2) = 190$, which is lower than the total cost 290 that would arise in the absence of trading. In this example, it is also easy to see that the advancement algorithm solves the cost-minimization problem (1)-(4). Thus, there is no need for retreat.

We can alter the above example in a simple way to show when retreat would be necessary. Continue to assume that both polluters are initially required to abate $R(1) = R(2) = 100$ kilograms, and that the trading ratios are $T(1,2) = 5/4$ and $T(2,1) = 4/5$. Now polluter 1 has two abatement steps. His first step retains the attributes $A(1,1) = 100$ and $C(1,1) = 100$ and his second step has attributes $A(1,2) = 150$ and $C(1,2) = 200$. Polluter 2 continues to have two abatement steps available. However, her first step now has attributes $A(2,1) = 900$ and $C(2,1) = 900$ and her second step has attributes $A(2,2) = 100$, and $C(2,2) = 1,000$.[8]

ITERATION 1: As in the previous example, $S_0(1) = S_0(2) = 0$, $P_0(1,2) = P_0(2,1) = 0$, and $F(1, S_0, P_0) = F(2, S_0, P_0) = 100$. Again by (5) – (7), pol-

[8]As we will see in the next section, this disparity in abatement levels and costs between polluters 1 and 2, respectively, reflects the actual disparities found for nonpoint and point sources in the Cub River sub-basin.

luter 1's effective average abatement cost is $EAC(1, S_0(1)+1) = 1$. However, polluter 2's effective average abatement cost is now

$$\begin{aligned} EAC(2, S_0(2)+1) &= \frac{C(2, S_0(1)+1)}{F(1, S_0, P_0) + \frac{A(2,S_0(2)+1) - F(1,S_0,P_0)T(2,1)}{T(2,2)}} \\ &= \frac{900}{100 + \frac{900-100\cdot(4/5)}{1}} = \frac{45}{46}. \end{aligned}$$

In this case, seller $i^* = 2$ again sells $F(1, S_0, P_0)T(2, 1) = 100\cdot(4/5) = 80$ to purchaser 1, and this time sells remnant $A(2, S_1(2)) - F(1, S_0, P_0)T(2, 1) = 900 - 80 = 820$ to herself. As a result, seller $i^* = 2$ now has an abatement surplus of $820 - 100 = 720$. The advancement algorithm's iterations end here, resulting in a total abatement cost of 900, which is only slightly lower than the total cost of $C(1, 1) + C2, 1) = 1,000$ that would arise in this new scenario in the absence of trading. A need of retreat is therefore apparent given polluter 2's large abatement surplus and the relatively small overall cost savings.

To begin the retreat process, consider the alternative of polluter 1 taking the first abatement step in the first iteration, rather than polluter 2. In this first step, $A(1, 1) = 100$ shrinks to $F(2, S_0, P_0)/T(1, 2) = 100 \cdot (4/5) = 80$ if polluter 1 sells to polluter 2. No remnant remains for polluter 1. Without loss of generality, we can assume that this is what polluters 1 and 2 agree to in the first iteration.

Now in the second iteration, it is clear that if polluter 2 abates (using what would be her first abatement step) a large surplus will again result. Thus, consider polluter 1's second abatement step $A(1, 2) = 150$ at cost $C(1, 2) = 200$. Of the 150 kilograms abated, polluter 1 can sell 25 to polluter 2, which shrinks to $F(2, S_1, P_1)/T(1, 2) = 25 \cdot (4/5) = 20$. This satisfies constraint (4) with equality for polluter 2 and leaves $150 - 25 = 125$ for polluter 1, which satisfies (4) with inequality, i.e., with a 25 kilogram surplus. Given the discreteness of the problem, this is the lowest surplus possible.

Therefore, using the retreat algorithm polluter 2 abates nothing while polluter 1 abates $\sum_{k=1}^{S(2)} A(1,k) = 250$ kilograms. The resultant total cost is $\sum_{i=1}^{2}\sum_{k=1}^{S(i)} C(i,k) = C(1,1) + C(1,2) = 300$, which is much lower than the total cost of 1,000 that would arise in the absence of trading, as well as the total cost of 900 that would arise using solely the advancement algorithm.

4 The Cub River

The Cub River sub-basin (to which the companion algorithms described in Section 3 will now be applied) is located in the Bear River Basin, Utah, which comprises 19,000 square kilometers of mountain and valley lands located in northeastern Utah (44 percent of watershed), southeastern Idaho (36 percent), and southwestern Wyoming (20 percent) – see Figure 1. The basin ranges in elevation from 1,283 meters to over 3,962 meters and is entirely enclosed by mountains. Agricultural lands throughout the basin, as well as urban areas, are located in valleys along the main stem of the Bear River and its tributaries. Common crops include dryland and irrigated pasture, hay, alfalfa, and corn, which are used locally to feed cattle and dairy cows.[9]

Currently, several water bodies in the basin are on the Clean Water Act's 303(d) list of impaired waters in each of the three states. One of these 303(d)-listed water bodies - the Cub River - forms the focus area for

[9]Utah DEQ [21] provides a detailed description of the basins physical, biological, and socio-economic characteristics. Total population in the basin is roughly 100,000 (U.S. Census Bureau [20].

this study (see Figure 2, which identifies receptor points for both the Cub River's confluence with the Bear River and Cutler Reservoir). The Cub River is included on the 303(d) list because of dissolved oxygen depletion during summer months due primarily to excessive total phosphorus (TP) loadings from point and nonpoint sources. A TMDL is currently being developed for the Cub River, and WQT has been identified as one potential solution to the TP problem.

From its point of entry in Utah, the Bear River and most of its tributaries flow through agricultural lands. As a result, major anthropogenic sources of TP loadings in the basin are nonpoint sources. The Cub River subbasin is comprised of approximately 1300 farms, four city-owned wastewater treatment plants (WWTPs), and one ice cream manufacturer. Aggregate annual delivered loads to the Cub River receptor point from point and nonpoint sources are estimated to be roughly 4,500 and 1,700 kilograms, respectively.[10]

5 An Application of the Trading Algorithms

To apply the advancement and retreat algorithms to the Cub-River subbasin, field-level data for nonpoint sources and source-level data for point sources were obtained for current loads (of total phosphorus), as well as $R(i)$, $A(i,k)$, $C(i,k)$, and $T(i,j)$, for each $i,j \in N$ and each k for each respective i.[11] This information was then 'fed' to the algorithms to generate values for

[10] According to Utah DEQ [21], loadings from animal feeding operations (both confined and unconfined) are already (or are in the process of becoming) adequately controlled through a variety of state- and federally funded programs. We therefore ignore loadings from these sources for the purposes of this study.

[11] The field-level data for nonpoint sources was then aggregated to the farm-level.

$P(i, j)$ and $F(i, S, P)$.

Total phosphorus (TP) load estimates for each nonpoint source located in the sub-basin were obtained from a water quality model developed by Neilson et al. [15]. Annual nonpoint-source TP loads range from approximately 52 to 0.002 kilograms. The Neilson et al. model also reports estimates of the corresponding trading ratios $T(i, j)$ for both point and nonpoint sources in the sub-basin. The estimated ratios range from 0.9 to 1.37. Point-source TP load estimates were obtained from those reported in Caplan et al. [4]. Annual loads range from approximately 1,700 kilograms (for a WWTP) to 0.001 kilograms (for the ice cream manufacturer).

For the purposes of this study, it is assumed that the target load for each nonpoint source is 30 percent of its current load, implying that $R(i)$ for all nonpoint sources is 70 percent of current loads. For all point sources, the associated $R(i)$ is 80 percent of current loads.[12]

Estimates for best-management-practice (BMP) effectiveness and per-acre costs for nonpoint sources were also taken from Caplan et al. [4]. Two types of BMPs were considered in that study – conservation tillage and nutrient management. We randomly assigned draws from the respective ranges of these estimates to each of the nonpoint sources in our population according to a set of pre-determined proportions, or probabilities. The values for BMP effectiveness (denoted as E), cost per-acre cost (denoted C/L), and associated probabilities for E and C/L, denoted π_E and π_{CL}, respectively, are included in Table 1.[13] For example, each nonpoint source in our popula-

[12] These percentages are in line with the preliminary Total Maximum Daily Load (TMDL) for the Cub River published by Utah DEQ [21]. For simplicity we assume uniform required reductions across all nonpoint and point sources, respectively.

[13] In relation to our previous use of notation, E multiplied by current load equals a nonpoint source's

tion had a 20 percent chance of being assigned a C/L of 3 and a 20 percent chance of being assigned an E of 60 percent, etc. In all, there were 3 x 3 = 9 possible combinations of C/L and E (with associated joint probabilities) for each nonpoint source.

Estimates for abatement levels and associated total costs for the sub-basin's point sources were also taken from Caplan et al. [4]. These estimates correspond to two "tiers" of abatement technology for each source. For Tier 1 technology, annual abatement levels (i.e., $A(i,1)$) range from 827 to 0.0003 kilograms; similarly for Tier 2 technology. Associated Tier 1 abatement costs (i.e., $C(i,1)$) range from \$344,000 to \$6,500, and Tier 2 costs (i.e., $C(i,2)$) range from \$362,000 to \$26,400.

Similar to the second part of the example presented in Section 3.2, where large differences in abatement levels and costs exist between the polluters, retreat was necessary to reduce both the sub-basin's abatement surpluses (i.e., the $F(i,S,P)s$) and its overall abatement costs to their lowest possible levels. In this respect, the sub-basin's nonpoint(point) sources are reminiscent of polluter 1(2) in the second part of the example in Section 3.2.

Due to the sheer number of the sub-basin's point and nonpoint sources involved in the trading exercise (as mentioned in Section 4 there are 1300 nonpoint sources and five point sources), we are precluded from presenting the final results for the advancement and retreat algorithms for each source (i.e., the trade pattern for the entire sub-basin). These results are available online at [TO BE INCLUDED UPON PUBLICATION]. Nevertheless, to

abatement level ($A(i,k)$), where $k=1$ for all i. The numerator in C/L is $C(i,k)$, again with $k=1$ for all i. L is the corresponding number of acres. Unlike for E and C/L, empirical estimates of π_E and π_{CL} do not exist in the literature.

get a flavor of the sub-basin-wide results, we highlight the results for two nonpoint sources (the two largest nonpoint polluters in the sub-basin) and two point sources (both WWTPs) that were involved in trades with each other as well as with other sources. In concert with the sub-basin-wide cost savings associated with the advancement and retreat algorithms (reported further below), these results demonstrate our main finding – that it can be cheaper for point sources to sell abatement credits to nonpoint sources as part of an overall WQT market outcome.

Profiles of the two point and two nonpoint sources are provided in Table 2. The profiles include estimates of current and target loads, required abatement ($R(i)$), abatement levels ($A(i,k)$), and total abatement costs ($C(i,k)$) for each source. For each of the two nonpoint sources (NPS_1 and NPS_2), only one abatement step (and associated cost) is available, while for each of the point sources (PS_1 and PS_3) two abatement steps are available.

As a result of running the trade algorithms on the entire sub-basin dataset, we obtain the following results for these four sources. NPS_1 abates nothing and instead purchases 39 credits (measured in kilograms) from PS_1 and 0.001 credits from another nonpoint source (classified as NPS_{85} in our dataset). Adjusting these credits via the corresponding trading ratios for these respective sources results in NPS_1 satisfying constraint (4) of Section 2 with equality, i.e., NPS_1 carries neither an abatement surplus nor deficit. In contrast, NPS_2 abates its full amount and sells *all* of its abatement – not just what would be its available credits if it retained enough of its abatement to satisfy constraint (4) – to PS_3. To meet its required reduction, NPS_2 then purchases enough credits (29 kilograms worth) from PS_1 to satisfy its constraint (4) with equality.

As for the two point sources, PS_1 abates its full amount (using both of its abatement steps) and sells all of its abatement to a large proportion of the sub-basin's nonpoint sources. To meet its required reduction, PS_1 then purchases 651 credits from another point source (classified as PS_2 in our dataset), 409 credits from PS_3, and the remainder from a host of nonpoint sources. In the end, PS_1 carries a negligable credit surplus. Similar to PS_1, PS_3 also chooses to abate its full amount (using both of its abatement steps). However, PS_3 sells its abatement to far fewer nonpoint sources than does PS_1, choosing instead to sell 409 and 356 credits to PS_1 and PS_2, respectively. PS_3 then purchases enough credits from a host of nonpoint sources to meet its own required-abatement constraint with equality.

Total annual abatement cost for the sub-basin to meet required TMDL load reductions under a no-trading scenario amounts to roughly \$2,706,000. In this scenario, approximately 80 percent of the nonpoint sources are able to meet their required reductions through their individual abatement efforts, and thus have surplus abatement available to 'cover' the deficit resulting from the remaining 20 percent that are unable to meet their requirements on their own. The net surplus amounts to 79.3 kilograms. Point sources would each need to implement both of their technology tiers to meet their required TP reductions. Similar to the nonpoint sources, this results in an abatement surplus. The surplus for point sources amounts to 740.7 kilograms. Thus, the total annual abatement surplus in the no-trading scenario amounts to approximately 820 kilograms.

In contrast, use of the advancement and retreat algorithms result in a total annual abatement cost for the sub-basin of roughly $1,780,000, i.e., a $926,000 savings relative to the no-trading scenario. Further, the abatement surplus for the entire sub-basin is a negligable amount. Therefore, in sum, the trading algorithms result in a substantial cost and abatement savings for the sub-basin by encouraging the widest possible gamut of trading opportunities – point-to-point, nonpoint-to-nonpoint, and bi-directional nonpoint-to-point.

6 Concluding Remarks

This paper has applied Sasaki and Caplan's [17] companion trading algorithms (the advancement and retreat algorithms) to a water quality trading (WQT) market located in the Cub River sub-basin of Utah; a market with both point and nonpoint sources. By accounting for all possible point-to-point, nonpoint-to-nonpoint, and bi-directional point-to-nonpoint source trades, the algorithms have been able to fully characterize the market's performance. Contrary to the commonly held belief that relatively high point-source abatement costs necessitate nonpoint-source abatement effort, we find that in a WQT market where each source is required to reduce its pollution loadings it is cheaper for at least some point sources to sell abatement credits to nonpoint sources as part of the sub-basin's overall trading outcome. This result occurs because although point sources do face relatively high total costs of abatement, on an average- or marginal-cost basis (which is the basis upon which potential trading is ultimately made) point sources instead face relatively low costs. Further, the abatement levels achieved by point sources (and thus the potential for these sources to generate abatement credits) can far exceed what disparate nonpoint sources are capable of achieving on their own.

The algorithms can easily be applied to a wide variety of WQT markets, and can be used by regulators to establish benchmarks – both at the watershed and individual-source levels – to both guide actual trades among sources and to assess the optimality of market outcomes over time. Indeed, the obvious deficiency of this paper is that it represents a single application of the trading algorithms to a single sub-basin in a single state. A welcomed extension of this paper would therefore be additional applications to a wide variety of WQT markets.

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C/L	π_{CL}	E	π_E
3	0.20	0.60	0.20
7	0.60	0.75	0.60
17	0.20	0.90	0.20

Table 1: BMP Effectiveness and Per-Acre Cost Information.

Source (i)	*Current Load* (kgs)	*Target Load* (kgs)	$R(i)$ (kgs)	$A(i,1)$ (kgs)	$C(i,1)$ (\$)	$A(i,2)$ (kgs)	$C(i,2)$ (\$)
NPS_1	52.1	15.6	36.5	39.1	8,530	–	–
NPS_2	38.6	11.6	27.0	28.9	2,692	–	–
PS_1	1,719.7	337.5	1,350.0	810.0	343,846	810.0	361,514
PS_3	1,722.1	344.4	1,377.7	826.6	303,385	826.6	303,473

Table 2: Profiles of Two Nonpoint Sources and Two Point Sources.

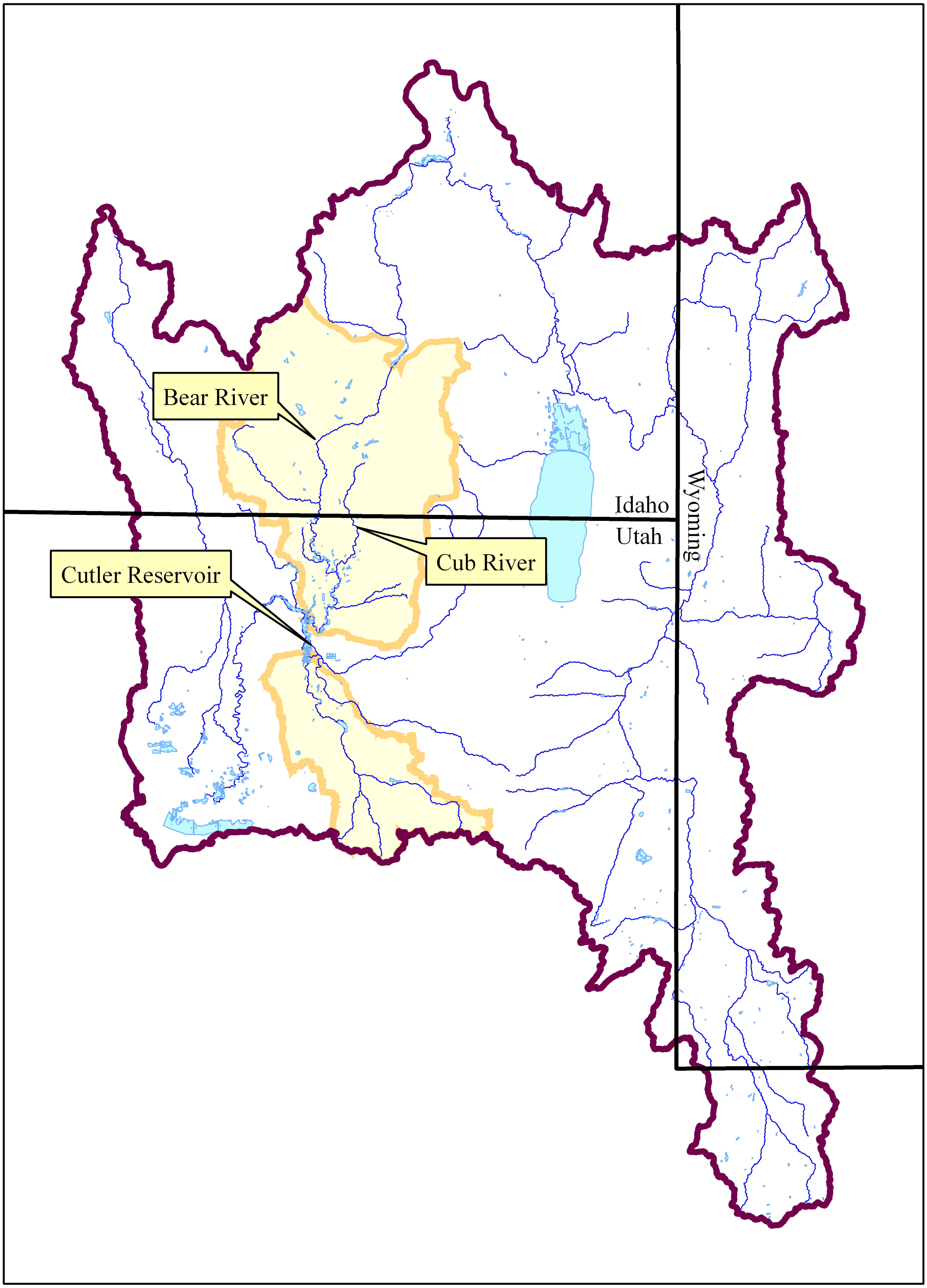

Figure 1: The Bear River Basin.

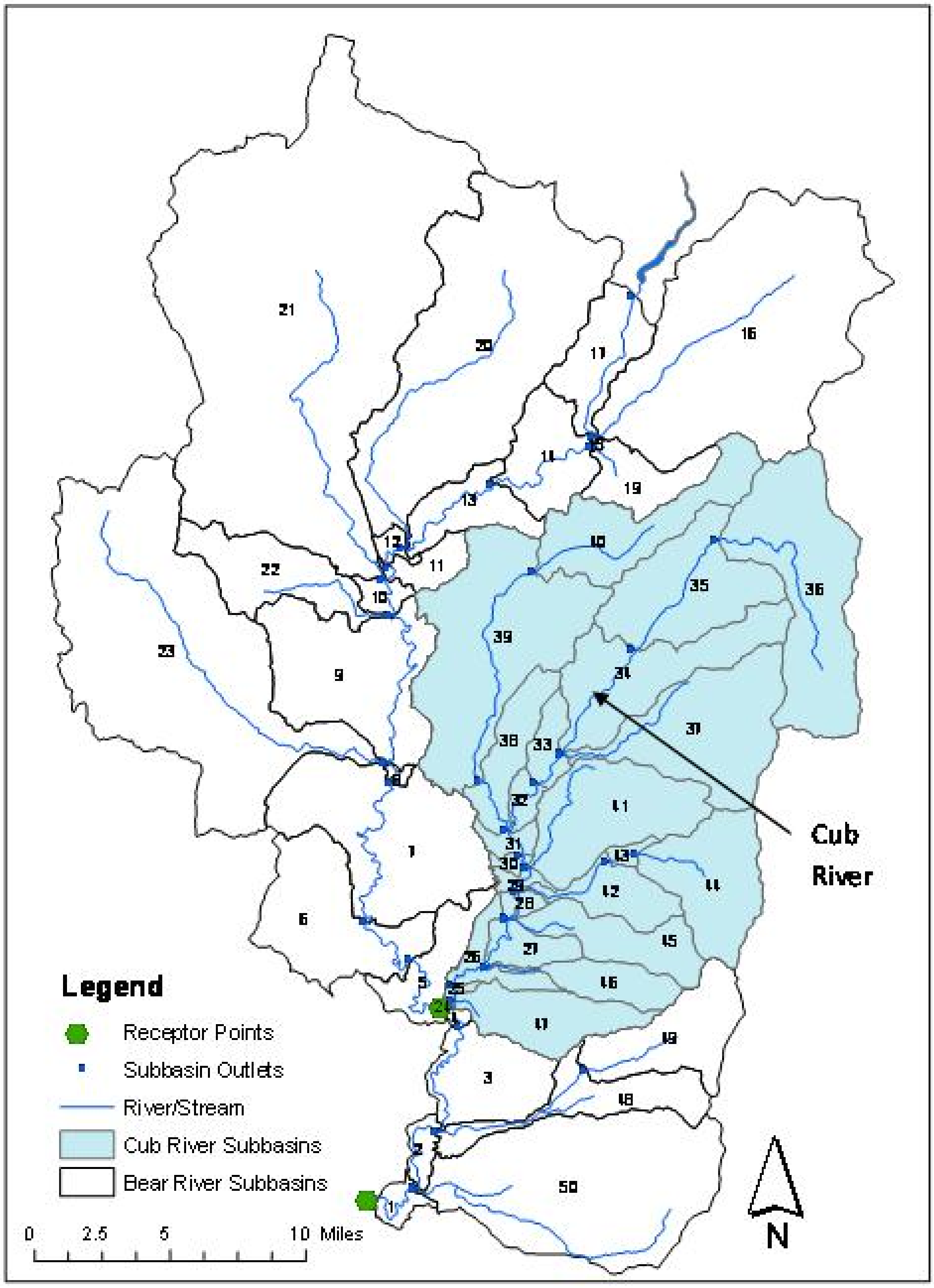

Figure 2: The Cub River Sub-Basin.